Using MATHEMATICA to Animate the Generation of a Space Centrode in Kinematics

R. R. Reynolds and I. C. Jong
University of Arkansas

Abstract

The software package MATHEMATICA provides a highly interactive computing environment in which scientific and engineering problems can be solved and the solutions displayed in a variety of ways. As such, it provides an efficient and concise means of solving and animating the motion of mechanisms. As part of an overall effort to provide multimedia tools for modern teaching, this paper uses some basic commands in the MATHEMATICA environment for animating a specific mechanism. The paper provides a complete program listing. A sequence of sample animation frames is included to illustrate the motion of a four-bar linkage and the plotting of the space centrode of its coupler.

Introduction

The study of the dynamics of mechanisms typically requires the student to visualize how a series of static configurations evolves in time. This requirement involves an intuitive leap that can be problematic for some people. This potential problem can be minimized or avoided by using an animation of the mechanism as a teaching tool. Unfortunately, animations have typically been difficult to create. The prevalence of commercial analysis software makes the creation and display of graphical aids much simpler now than it has been in the past.

MATHEMATICA[1] allows users to create programs using a high-level programming language and to manipulate and display data easily. This software provides an extensive library of subroutines, which can be used to solve problems with a minimum of programming effort. These subroutines include numerical integration, root-finding, and curve fitting. It is an excellent package for seamlessly integrating numerical computations, data analysis, and presentation. The graphics capabilities of MATHEMATICA include two and three dimensional curve and surface graphing, density plots, and the ability to create a wide range of primitive graphics entities (e.g., lines, circles, arcs, etc.). In addition to these capabilities, the program interface permits the user to animate any sequence of graphics entities. Thus, by simply generating a series of images that depict the motion of the linkage mechanism and its velocity center over a range of positions, we may animate the motion of the linkage as if it were moving in real-time by issuing the *animate* option.

Recently Jong and Onggowijaya[2] used QuickBASIC to animate the generation of the space centrode of a coupler as a complement to WORKING MODEL. Adams and Jong[3] also animated the space centrode using the MATLAB software. For the purpose of comparison, the generation of the same space centrode is solved and animated here using the MATHEMATICA program. A selection of sample animation frames is included in Appendix A, and a listing of the complete yet brief MATHEMATICA program used to create this animation is given in Appendix B, where various helpful comments are inserted as comments between the marks `(*` and `*)`.

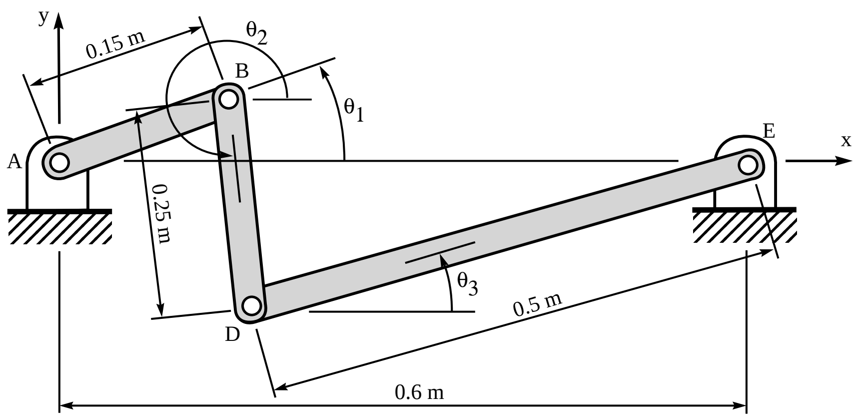


Figure 1 A four-bar linkage with no lockup positions.

Four-Bar Linkage

The linkage mechanism employed herein has a full range of motion (i.e. no lockup positions) and is depicted in Fig. 1, where the input crank is rotated with an angular velocity

$$\omega_{AB} = \dot{\theta}\boldsymbol{k} \text{ rad/s} \tag{1}$$

Jong et al.[4] showed that the constraint equations for this linkage are

$$\overline{AB}\cos\theta_1 + \overline{BD}\cos\theta_2 + \overline{DE}\cos\theta_3 = \overline{AE} \tag{3}$$

$$\overline{AB}\sin\theta_1 + \overline{BD}\sin\theta_2 + \overline{DE}\sin\theta_3 = 0 \tag{2}$$

where $\overline{AB}$ = 0.15 m, $\overline{BD}$ = 0.25 m, $\overline{DE}$ = 0.5 m, and $\overline{AE}$ = 0.6 m. To determine the linkage configuration must be solved for and for each input crank angle θ_1.

Since the equations are transcendental, an iterative root finding algorithm must be employed. The MATHEMATICA subroutine library includes a routine for solving nonlinear, simultaneous equations using either Newton's method or the secant method. The form of the command is

```
FindRoot[{equation 1==0, equation 2==0,...,equation 3==0},
     {θ2,θ2i}, ... , {θn,θni}]
```

where θ_n is given an initial value θ_{ni}. We use this subroutine by inputing Eqs. (2) and (3) and initial values for θ_2 and θ_{2i} into the above form of the `FindRoot` command. Appendix B includes an unabridged listing of the MATHEMATICA program for solving and generating the animation.

The velocity center ***C*** of the coupler link ***BD*** is the point of intersection of the lines ***AB*** and ***DE***. Its coordinates can be shown to be given by

$$x_c = -\frac{\overline{AE}\tan\theta_3}{\tan\theta_1 - \tan\theta_3} \quad \textbf{(4)}$$

$$y_c = -\frac{\overline{AE}\tan\theta_1\tan\theta_3}{\tan\theta_1 - \tan\theta_3} \quad \textbf{(5)}$$

This location is plotted at each incremental value of θ_1 and the cumulative locus of the velocity center (i.e., the space centrode) drawn in a separate color.

To show the full range of motion of the linkage, the input crank must complete two full revolutions. Thus, Eqs. (2) through (5) are solved at discrete positions in the range $0 < \theta_1 < 720^\circ$. MATHEMATICA allows this to be done with a simple conditional loop. In this case, a **`While`** loop is used in the form

```
While[ θ1 < (θ1)max,
      FindRoot[ ... ];
      xc = ...;
      yc = ...;
      θ1 += Δθ1 ]
```

Once the solutions are determined, they are used to create a series of plots depicting the linkage configuration and the velocity center for each incremented value of θ_1. Naturally the memory of the computer being used is a limiting factor in the number of images that can be generated and animated. The powerful graphics capability of MATHEMATICA allows the user to select many of the parameters related to these images. These selections include

- the size and color of the points drawn;
- the width, style, and color of the lines drawn;
- the axes styles, labels, aspect ratio, as well as the range of the values plotted.

After the images are drawn, a user may select (i.e., "click" on) the complete set of images and use the *Animate* option on the user interface to have the images presented in rapid succession. The speed of the animation may be adjusted *a priori* or while the animation is occurring via option buttons displayed on the user interface.

Results

An animation of the generation of the space centrode of the coupler for a four-bar linkage has been created. Selected sample frames from this animation are included in Appendix A. These images display the linkage bearings at (0,0) and (0.6,0), the three moving links, the velocity center, and the space centrode as it has evolved until that instant. The progression of the animation reveals the following features, which are displayed in Appendix A:

1. For $0^\circ < \theta_1 < 22.5^\circ$, the space centrode lies in the first quadrant and extends outward to an increasing distance from the mechanism (cf. Frame a in Appendix A).

2. For $22.5^\circ < \theta_1 < 23.5^\circ$, the space centrode jumps from the first quadrant to the third quadrant tracing an asymptote that indicates the existence of a configuration in which the input and output links are parallel (cf. Frames b and c).

3. For $23.5^\circ < \theta_1 < 336.5^\circ$, the space centrode approaches the origin, passes around it, and moves away in the second quadrant (cf. Frames d and e).

4. For $336.5^\circ < \theta_1 < 337.5^\circ$, the space centrode jumps from the second quadrant to the fourth quadrant tracing an asymptote that indicates the existence of a configuration in which the input and output links are again parallel (cf. Frames f and g).

5. For $337.5^\circ < \theta_1 < 360^\circ$, the space centrode moves closer to the mechanism and passes through the bearing support for the output link (cf. Frames h and i).

6. For $360^\circ < \theta_1 < 720^\circ$, the space centrode evolves counterclockwise into a closed path encompassing the input link. The linkage completes the cycle by returning to the starting configuration (cf. Frames j, k, and l).

Conclusions

To aid students in their understanding of the kinematics of a mechanism, the MATHEMATICA program was used to solve the mechanism constraint equations and to generate a sequence of frames depicting the moving four-bar linkage and the location of the velocity center of its coupler. These frames were then animated to help students understand how the space centrode evolves during the motion of the linkage.

Although MATHEMATICA is not the only software package available for this purpose, it has the advantages of being able to solve the equations very easily and to generate a clear animation with a minimum of commands. Furthermore, MATHEMATICA is increasingly available and familiar to many students and teachers.

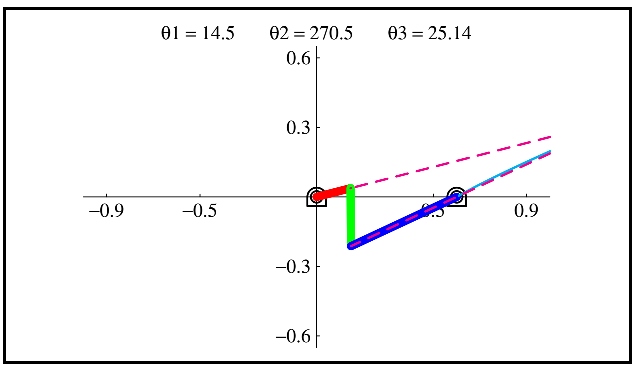
References

1. Wolfram, S, *The Mathematics Book*, Wolfram Media, Champaign, IL, 3rd edition, 1996.
2. Jong, I.C, and Onggowijaya, S.N., "Animation Programming with QuickBASIC to Aid the Teaching of Kinematics," to be published.
3. Adams, G.P., and Jong, I.C., "Using MATLAB to Animate the Generation of a Space Centrode in Kinematics," to be published.
4. Jong, I.C., Reynolds, R.R., and Adams, G.P., "Determination of the Space Centrode of a Coupler Link," *1996 Annual ASEE Conference Proceedings*, session 2668, Washington, DC, June 23-26, 1996.

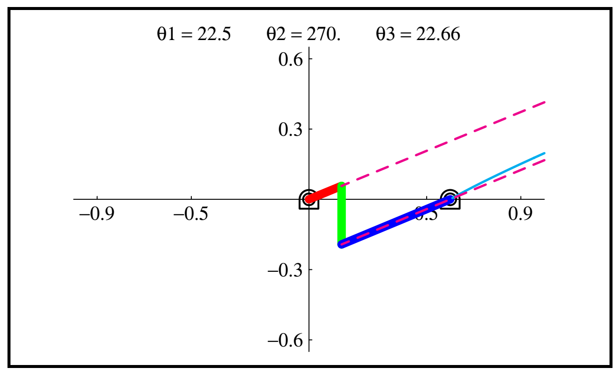
R. R. Reynolds
Received his BSME from Carnegie-Mellon University in 1985. He worked as a design engiineer for Data General Corp. from 1985 until 1987 when he began graduate school. He received his MSME from Purdue University in 1989 and his Ph.D. from Duke University in 1993 specializing in nonlinear dynamics and aeroelasticity. After a 1 year postdoctoral position, he joined the faculty of the University of Arkansas as an Assistant Professor of Mechanical Engineering.

I.C. Jong
Ing-Chang Jong received a BSCE from the National Taiwan University in 1961, an MSCE from the SDSM&T in 1963, and a Ph.D. from Northwestern University in 1965. He is Professor of Mechanical Engineering at the University of Arkansas. He and Dr. Bruce Rogers published an engineering mechanics textbook in 1991. He is serving as the Chair of the Mechanics Division, ASEE, 1996-97.

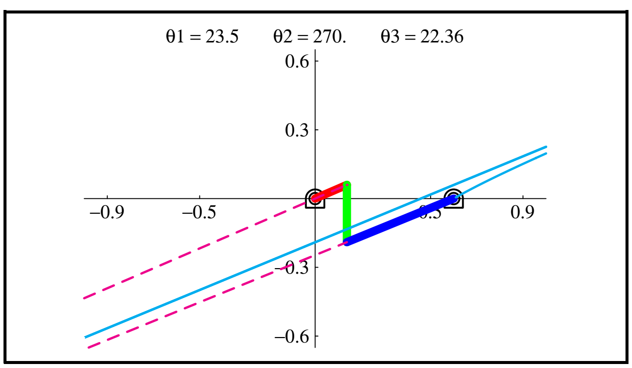
Appendix A: Sample Animation Frames



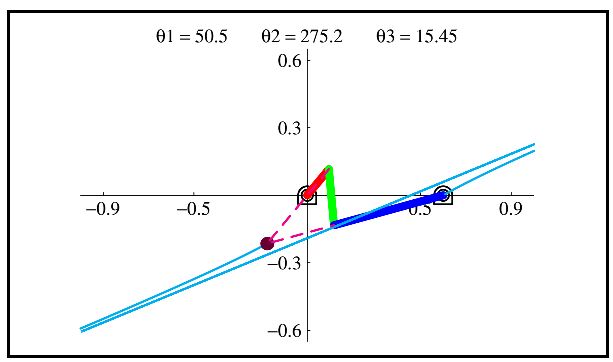
(a) $\theta_1 = 14.5^\circ$



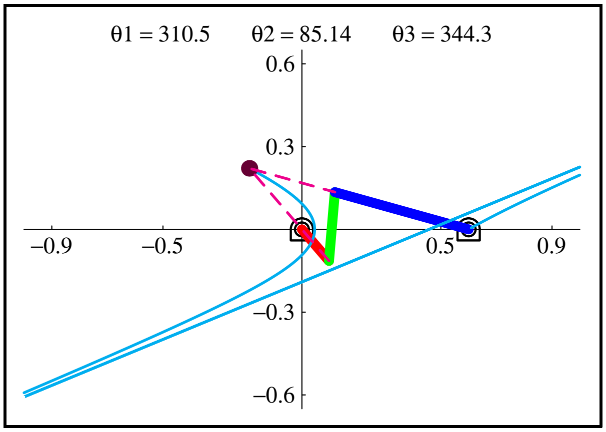
(b) $\theta_1 = 22.5^\circ$



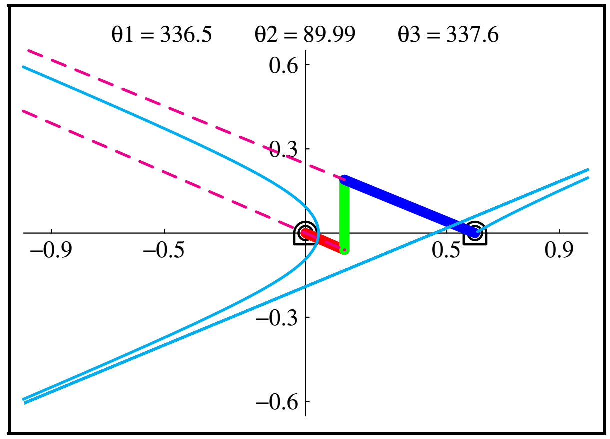
(c) $\theta_1 = 23.5^\circ$



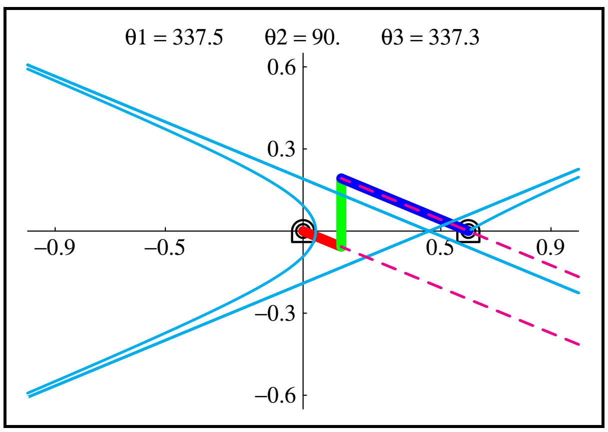
(d) $\theta_1 = 50.5^\circ$



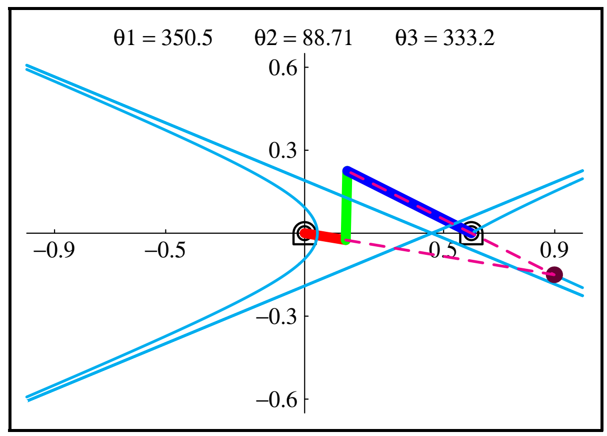
(e) θ_1=310.5°



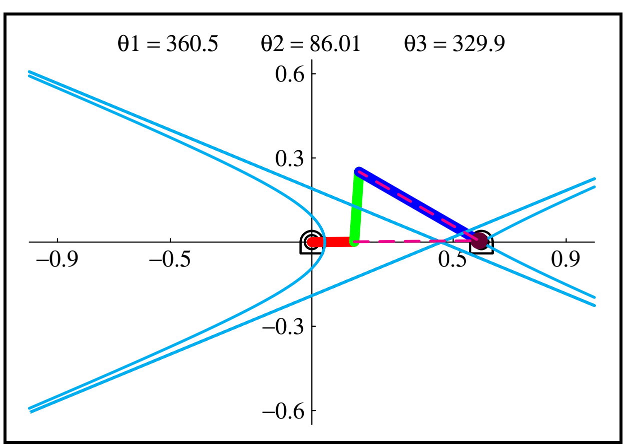
(f) θ_1=336.5°



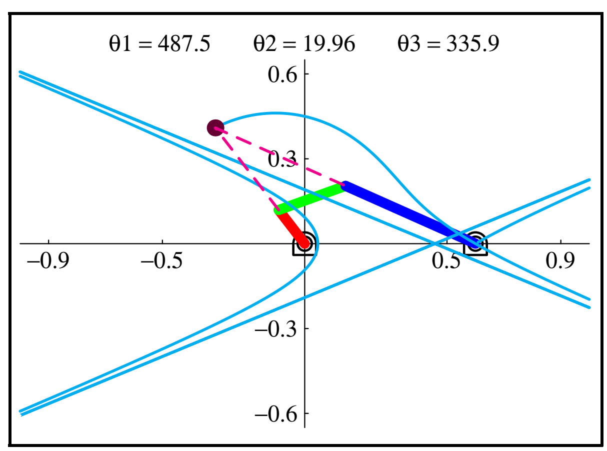
(g) θ_1=337.5°



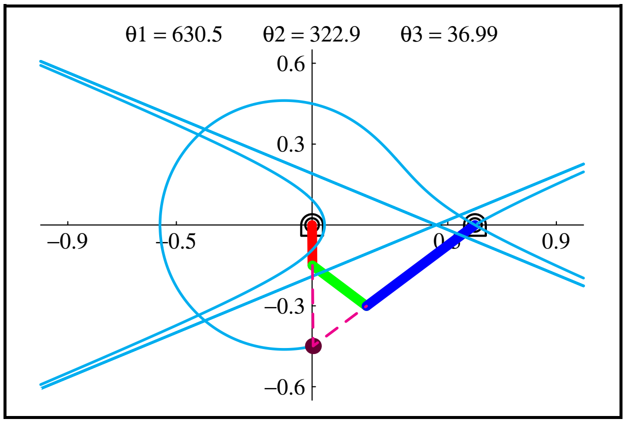
(h) θ_1=350.5°



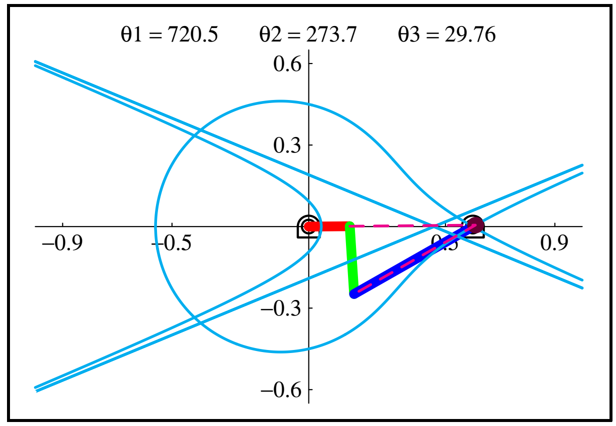
(i) θ_1=360.5°



(j) θ_1=487.5°



(k) θ_1=630.5°



(l) θ_1=720.5°

Appendix B: MATHEMATICA Program Listing

```
(*
            PROGRAM TO ANIMATE A 4-BAR LINKAGE and
            SHOW SPACE CENTRODE
======================================================================*)
(*  Initialize the variables used to solve the simultaneous equations *)
theta = {};
theta1 = N[1 Degree];
theta2init = 270 Degree;
theta3init = 30 Degree;
theta1max = N[2 * 360 Degree];
deltatheta = N[ 2 Degree ];
i = 1;
(*-------   Loop where simultaneous equations are solved   -------*)
While[  theta1 <= theta1max
   AppendTo[ theta
      FindRoot[ (* actual solution subroutine - Newton's method *)
         {0.15*Sin[ theta1 ] + 0.25*Sin[ theta2 ] + 0.5*Sin[ theta3 ] == 0
         0.15*Cos[ theta1 ] + 0.25*Cos[ theta2 ]+ 0.5*Cos[theta3 ]  == 0.6},
         {theta2, theta2init}, {theta3, theta3init} ]];
      While[ theta[[ i,3 ]] < 0, theta[[ i,3 ]]+= N[2 Pi] ];
      theta1 += deltatheta;
      theta2init = theta[[ i, 2 ]];
      theta3init = theta[[ i, 3 ]];
      localtheta = N[ Mod[  theta1, 2 Pi ] - Pi ];
      If[ Sign[ localtheta * (localtheta-deltatheta) ] == -1,
         theta2init = N[ Mod[ -2*theta2init, 2 Pi] ] ];
         i += 1 ];
(*-----------   CENTRODE   CALCULATION  --------------*)
velcenter = {};
Do[  (* loop over solution array to calculate centrode *)
   xc = 0.6*Tan[theta[[i,3]]] / (Tan[theta[[i,3]]] - Tan[theta[[i,1]]]);
   yc = xc*Tan[ theta[[ i, 1 ]];
   AppendTo[ velcenter, {xc, yc} ],
      {i, Length[theta]} ];
(*--------------  Create the Animation Graphics  ---------------*)
joints = {};
Table[ (* calculate the joint locations *)
   AppendTo[ joints, {
      {0.15*Cos[ theta[[i,1]] ], 0.15*Sin[ theta[[i,1]] ]},
      {0.6-0.5*Cos[ theta[[i,3]] ], -0.5*Sin[ theta[[i,3]] ]} }
   ], {i, 1, Length[theta]} ];
anim = Table[{ (* create the array of animation graphics *)
```

```
   Circle[ {0,0}, 0.025], Circle[{0,0}, 0.04, {0, 180 Degree}],
      Line[ {{-0.04,0}, {-0.04,-0.04},{0.04,-0.04},{0.04,0}}],
   Circle[ {0.6,0}, 0.025}, Circle[ {0.6,0}, 0.04, {0,180 Degree}],
      Line[{ {0.6-0.04,0}, {0.6-0.04,-0.04}0.6+0.04,-0.04},{0.6+0.04,0}}],
   Thickness[0.018], RGBColor[1,0,0], Line[{{0,0}, joints[[i,1]] }],
      RGBColor[ 0, 1, 0], Line[{joints[[i,1]], joints[[i,2]] }],
      RGBColor[ 0, 0, 1], Line[{joints[[i,2]], {0.6,0} }],
   Thickness[0.005], CMYKColor[ 1, 0, 0, 0], Line[ Take[ velcenter, i]],
   PointSize[ 0.03 ], RGBColor[ 0.4, 0, 0.2 ],
      Point[ {velcenter[[ i, 1 ]], velcenter[[ i, 2]] } ],
   CMYKColor[ 0, 1, 0, 0 ], Dashing[ { 0.025, 0.02}],
      Line[ { joints[[ i,1 ]], velcenter[[ i ]], joints[[i,2]] }]},
   { i, 1, Length[ joints ] } ];
(*--------------------   Now Plot  it   -----------------*)
Table[  Show[ Graphics[ anim[[ i ]] ], (* SHOW the anim. w/ parameters *)
   AspectRatio->0.75, Axes->True,    (* show axes and achieve equal
                                        scales on x and y axes *)
   Ticks->{{-0.8,-0.4,0.4,0.8}, {-0.6, -0.3, 0.3, 0.6},
   PlotRange->{{-0.8,0.8},{-0.6,0.6}},
   PlotLabel-> StringForm[ "θ1 = `` θ2= `` θ3= ``",
      N[ theta[[ i,1 ]]/Degree, 4 ],
      N[ theta[[ i,2 ]]/Degree, 4 ],
      N[ theta[[ i,3 ]]/Degree, 4 ],
   DefaultFont->{"Symbol",12}
],{i,1,Length[anim]} ]
```