



Using Nonlinear Programming to Optimize the Fiber Packing Density of Optical Fiber Cables- A Short Problem-Based Learning Course

Dr. Kenneth W. Jackson, Southern Polytechnic State University

Kenneth W. Jackson, Ph.D. – P.E.

Dr. Ken Jackson received his Ph.D. in Mechanical Engineering from the Georgia Institute of Technology. He also holds an M.S.M.E and a M.S. I.E. from Georgia Tech and a B.S.M.E from Auburn University. Before joining SPSU he worked for 15 years at the Bell Laboratories as a Consulting and Distinguished Member of Technical Staff. At Bell Labs Dr. Jackson worked on the design, development and commercialization of fiber optic products for telecommunication systems and their associated manufacturing processes. Before joining Bell Laboratories he also worked for the Western Electric Company in defense activities, manufacturing and product engineering. Dr. Jackson's technical interests include statistical computing, applying innovative methods to improve industrial systems' performance and the development and application of system-structured design and development tools to reduce the cost and increase the quality of new products. Dr. Jackson has authored 17 patents and has published/presented 23 technical papers. He is a Registered Professional Engineer in Georgia, a member of the American Society for Quality and the ASME.

Prof. Gregory L. Wiles P.E., Southern Polytechnic State University

An assistant professor of industrial engineering technology at Southern Polytechnic State University, a four-year technical university in Georgia. He has a BS degree in Industrial Engineering at the University of Tennessee, an MS degree in Industrial & Systems Engineering at the Georgia Institute of Technology, and currently working on his PhD. Prior to teaching, he worked for Lockheed Martin, Union Carbide, nVision Global, Oracle, and Georgia Tech in various engineering roles from research, to technical sales, to division management.

**Using Nonlinear Programming
To Optimize the Fiber Packing Density
Of Optical Fiber Cables -
A Short Problem-Based Learning Course**

Abstract

A need for time-compressed, low credit hour, problem-based courses for upper division students frequently arises. These courses may be used to support universities' honors program by extending the learning mastery of traditional subjects through experiential learning and challenging the exceptional student. In addition, such courses can facilitate and allow some students to graduate on schedule. Although short, problem-based courses with low enrollment (2-5 students) do not map conveniently to the traditional teaching schedule of faculty, they can provide a unique opportunity for service both to students and to the university, especially in summer semesters. In addition, the National Academy of Engineering has recognized experiential learning which involves solving open-ended problems that are complex and ill structured as a critical competency for engineers in the 21st century.

This paper describes the design of a special problem course in which we use the case method to teach industrial engineering technology students nonlinear integer programming, while providing a collaborative learning experience in solving a simulated real world problem. The basic problem uses nonlinear integer programming to explore the optimization space for optical fiber ribbon-based cables, which are subject to a variety of practical constraints. Students learn to use the LINGO software package to maximize the fiber packing density for a number of exemplary cable configurations. Students also learn a variety of skills and acquire an intuitive understanding of optimization algorithms. The problem complexity and the scope of the course can be adjusted to serve the needs and abilities of the students. End-of-course deliverables include a formal engineering report and the preparation and presentation of a poster paper. The special problem course provides students with a collaborative learning experience solving an ill-structured practical problem and it provides faculty a unique service opportunity.

Keywords: Problem based learning, nonlinear optimization, honors program, non-linear integer programming

Introduction

We describe a time-compressed, low credit hour special problem course (1-2 credit hours) that can be offered either to individual students or to a very small cohort of students who share a common interest in a contemporary engineering/technological topic not typically covered in their regular coursework. For example, in a traditional introduction to operations research course engineering technology students are typically taught only linear programming, both integer and non-integer based. The linearity assumption for many modern applications in engineering, operations, business and economics does not accurately describe either the objectives or constraints. Thus, nonlinear programming (NLP) with integer or mixed decision variables has become an important tool for optimizing various engineering and business operations as well as for the optimization of physical designs. The availability of high quality, affordable and user-friendly software now makes it feasible to introduce students to NLP as a natural extension to LP in their operations research studies. In addition, nonlinear optimization may provide a more intuitive understanding of how an optimization algorithm works in contrast to the simplex algorithm, which is non-intuitive and can be conceptually difficult for the beginning or business student to understand⁷. We use a case study which involves maximizing the fiber packing density of fiber optic ribbon cables to teach students how to use nonlinear programming to solve a simulated real world problem with a variety of evolving constraints. Two professors *teach* the course in a non-lecture, collaborative mode to simulate better the collaborative communications used for problem solving in typical work environments and to reduce the distinction between teacher and learner¹. According to Ackoff et al., students are best motivated to learn and to learn how to learn when solving real problems under real conditions².

The focus of the case study involves maximizing the number of optical fibers that can be placed into a cable core tube having a fixed diameter⁸. The fibers are prepackaged into ribbons, thus this problem represents a simple subset of the more general and contemporary problem of optimally packing rectangles inside circles³. For many telecommunication applications, maximizing the fiber packing density minimizes the installed cost per fiber to the service provider. In addition, high fiber count cables may be needed for specialized applications. In order to facilitate mass splicing of optical fibers, which reduces installation costs, the fibers are pre-packaged into rectangular, ribbon-like arrays comprising variable fiber counts before placement into the cylindrical cable tube core in the manufacturing process. Figure 1 illustrates exemplary fiber ribbons comprising twelve, twenty-four and thirty-six fibers. The optimization may use linear and nonlinear constraints to restrict the probability of excess, added loss during cabling and to achieve stability and symmetry of the ribbon stack. Similar constraints may also exist between the global geometry of a complex ribbon stack and the circular cross-section of the cable core. The problem complexity can be evolved gradually to allow the students to discover and include new constraints and practical considerations of how to identify and remove infeasibilities that may arise as the nature of the problem changes.

This special problem course can provide students an effective learning experience solving a complex, ill-structured problem which the National Academy of Engineering has recognized as a critical competency for engineers in the 21st century⁴. Compared to traditional textbook problems, understanding a real problem is more time consuming and requires more inquiry and discovery by the students. Real problems also require more independence, initiative and patience on the part of the problem solver(s). Real world problems are not strongly disciplinary in the traditional sense and require the integration of several skills that students acquire in their program of study. Moreover, understanding the context and constraints of a real problem is a discovery process that unfolds gradually over time. Thus, real problems are loosely defined and are *not designed to be solved* like the textbook problem; they are what they are. Moreover, the body of knowledge necessary for the solution to real problems is not preassembled, but is distributed among a diverse group of subject matter experts. In this paper, the term ill structured refers to the open-ended nature of a problem having multiple competing goals where one must use judgment to make needed tradeoffs⁹.

Case description

The Optical Cable Systems Company, (OCSC) has been a leading supplier of optical fibers and cables to telecommunication service providers since the birth of the industry more than twenty years ago. OCSC's current annual revenue is very large, but the fiber cable technology is entering its mature phase where feature innovations are declining and the installed cost per fiber increasingly drives competition. OCSC has several potential customers who wish to install very high capacity optical cables in densely populated metropolitan areas. An important constraint on the cables in these applications is that they must be capable of being installed in underground conduits, with a fixed inside diameter, for example 1.25 inches. This conduit diameter may restrict the outside diameter of the cables to be less than one inch⁵. The installation and associated construction work involves excavation of city streets and is considerably disruptive for day-to-day commercial activities. The disruption factor may restrict installation intervals at a given site to ten or more years. Owing to the disruptions of commerce, the installation costs could be several times the cost of the cable itself. The foregoing issues, along with the potential for lost sales, make it desirable to design a family of cables that can accommodate a maximum number of fibers. The fibers may be prepackaged into rectangular geometry ribbons having a variable thickness and fiber count. A grease-like material is injected into the space between the ribbons and the core tube to further prevent ingress of water. The ribbon stack is twisted as it is placed into the core tube in order to improve bending performance and fiber loss stability under temperature extremes.

OCSC's marketing organization concludes that cables with a higher fiber packing density can command a greater profit premium than traditional cables whose fiber packing density is roughly 546 fibers per inch of core tube diameter. Marketing would like to create new designs that can at least double the traditional fiber packing density. Practical considerations normally militate against manufacturing an optimal cable core for a given fiber count so management has

mandated that only a few core tube sizes comprise the standard product line. However, management would like to generate a graph of the potential fiber count optimization space for several tube sizes initially in order to facilitate selection of the standard sizes. Notwithstanding a family of standard cable sizes, it may be profitable to provide a cable optimized specifically for a particular application.

Figure 1 shows representative dimensions of conventional fiber ribbons comprising coated fibers having a nominal outside diameter of 250 microns. Researchers are suggesting that new specialized coating materials may be able to reduce the coated fiber diameter to as low as 175 microns, which would allow a reduction in ribbon sizes and would potentially increase the fiber packing density further. However, the development horizon for the new coating material technology is highly uncertain and if used would also require more free space inside the core tube. The marketing organization, wishes first to study the potential fiber counts that might be achieved with the current standard 250 micron fibers. The ratio of the various ribbon stack diagonals, illustrated in Figures 2a and 2b, to the core tube diameter are critical-to-quality (CTQ) design parameters⁵. It has been empirically found that a specified d/D ratio for a given stack twist length is required to achieve good bending and added loss performance of the cables. The value of the d/D parameter can vary depending on the mechanical properties of the ribbon matrix material, the geometry, the fiber coating, and the twist lay-length of the ribbon stack. Business and manufacturing managers might consider making cables with increased d/D ratios and use an allowance for remakes for a given cable if the financials of the application justifies it.

The number of fibers in a cable is a linear additive function of the number of fiber ribbons of each fiber count, thus providing a linear objective function. We wish to maximize this objective function subject to constraining the probability, P, of making a defective cable due to excess added loss during cabling. The probability of making a defective cable has empirically been determined to be an increasing function of the ratio of the stack diagonal to the core tube inner diameter, d/D. Symbolically,

$$\text{Equation (1) } P = f(d/D)$$

where the empirically estimated probability, P, of making a defective cable is a function of a given d/D ratio.

Equation 1 assumes that other relevant variables such as the ribbon stack twist period, the ribbon excess length, and the loss sensitivity of the fibers are constant.

A constant d/D corresponds roughly to a constant probability of a defective cable. Specific values of the d/D ratio are determined from empirically and depend on several processing and design parameters. We selected d/D range of 0.5-0.9 for study of the optimization space.

Fixing both d/D and D corresponds to a fixed clearance between the stack diagonal and the inner wall of the core tube. This clearance in turn is also given by a nonlinear expression involving the

number of fiber ribbons of each fiber count and the individual ribbon dimensions. So different values of d/D for given D correspond to a nonlinear constraint among the decision variables and the higher the value of d/D the higher the likelihood of making a defective cable. An expression for the clearance can be obtained by using the familiar Pythagorean Theorem.

Management has assembled a team of product developers whose assignment is to explore the potential optimization space available for high fiber packing density cables. Your role [the student] on the team is to help develop optimization models for the cable designers. Your initial assignment is to generate optimization curves for the maximum fiber count that a cable with a given core tube diameter can accommodate as a function of the d/D ratio. The cables may comprise fiber ribbons having twelve, eighteen, twenty-four, or thirty-six fibers either singly, or in various combinations.

Basic problem formulation

For reference, Figures 2a and 2b show cable core tubes with exemplary ribbon arrangements. The primary decision variables in the initial problem are X_1 , X_2 , and X_3 , which represent the respective numbers of fiber ribbons comprising thirty-six, twenty-four, and twelve fibers to use in a given cable. Thus, following expression gives a typical objective function:

$$\text{MAX} = 36 * X_1 + 24 * X_2 + 12 * X_3$$

For illustration purposes, consider a core tube having an inside diameter, D of 0.520 inch. The first three constraints which follow below all assume the same d/D ratio of 0.815, which in turn gives a radial clearance of about 0.048 inch between the various diagonals and the core tube wall. In practical cases the d/D ratio will vary depending on the particulars for a given application where a higher d/D reflects a higher probability a given cable might require a remake. The clearance between a ribbon sub stack and the core tube wall may be calculated in terms of the foregoing decision variables by using the ribbon dimensions along with the familiar Pythagorean formula (in pseudo Lingo code).

- C.1 .26- ((X1*.00625)^2 + (.1875)^2)^5 - 0.048 >= 0
- C.2 .26- ((X1*.00625 + X2*.00625)^2 + (.125)^2)^5 - 0.048 >= 0
- C.3 .26- ((X1*.00625 + X2*.00625 + X3*.00625)^2 + (.0625)^2)^5 - 0.048 >= 0
- C.4 Y1=X1/2
- C.5 Y2=X2/2
- C.6 Y3=X3/2
- C.7 @GIN(Y1)
- C.8 @GIN(Y2)
- C.9 @GIN(Y3)
- C.10 @GIN(X1)
- C.11 @GIN(X2)
- C.12 @GIN(X3)

The decision variables are integers and the constraints involving Y require the number of ribbons in a cable to be even and arranged symmetrically on either side of the core tube centerline (These constraints can induce rather large jumps in the maximum fiber counts). The solution for this example is as follows:

Objective value: 864.000

Variable	Value
X1	14.000000
X2	14.000000
X3	2.000000

The formulation may be modified to include a different d/D for each ribbon sub-stack, different core tube diameters, and restrictions on the numbers of ribbon of a given type, including using only a single ribbon size or multiple ribbon sizes. These combinations can then be used to generate graphs like the one shown in Figure 3 depicting the maximum fiber count for a given core tube size and ribbon arrangement. Such graphs can also be used to solve the reverse problem: finding the minimum core tube size needed to accommodate a given number of fibers and specified ribbon types. Students discover that infeasibilities arise for some variable combinations and they then have to identify the offending constraint(s) and modify the code. Lastly, students may discover multiple combinations of variables that admit the same fiber count.

At the start of the course we give students a notebook with the case description, exemplary ribbon dimensions, and a fiber optics article describing the problem, relevant LINGO documentation, a patent reference and instructions for creating a poster paper. We describe roughly how the Pythagorean theorem can be used to derive an expression for the geometrical constraints in order to get students started on the optimization for the simplest case where a cable comprises ribbons that all have the same fiber count. All materials and dimensions are public information that is available on the Web. From this point we use subsequent meetings to evolve the problem further, to review deliverables from previous meetings and to coach students on additional work.

For completeness, near the end of the course, we cover briefly the Markowitz mean-variance portfolio optimization model, which is the classical problem paradigm for the application of nonlinear programming. In addition, we emphasize to students that there are several approaches to solving nonlinear optimization problems.

Conclusion

We have designed and prototyped a time-compressed, low credit hour, problem-based course for teaching engineering technology students nonlinear programming via a collaborative learning experience. The course is team-taught using a non-lecture case study format to better simulate a work environment. We use a simulated real world example, which involves maximizing the

fiber count of optical fiber ribbon cables. Students learn nonlinear programming, write a report, and prepare a poster paper and an oral presentation. At the end we cover briefly the Markowitz mean-variance portfolio optimization model, which is the classical problem paradigm for the application of nonlinear programming. This type course may be used to support universities' honors programs by extending the learning mastery of traditional courses and it can enable some students to graduate on schedule ⁶. The course can also provide a unique opportunity for faculty service to both students and the university. In addition, the course addresses the recommendation of The National Academy of Engineering for increased experiential learning through solving more open-ended problems that are complex and ill structured and which require collaboration and judgments among team members.

Disclaimer: The case study and calculations represent a hypothetical situation selected for educational purposes. Any resemblance to a specific fiber optical cable design or company is purely coincidental.

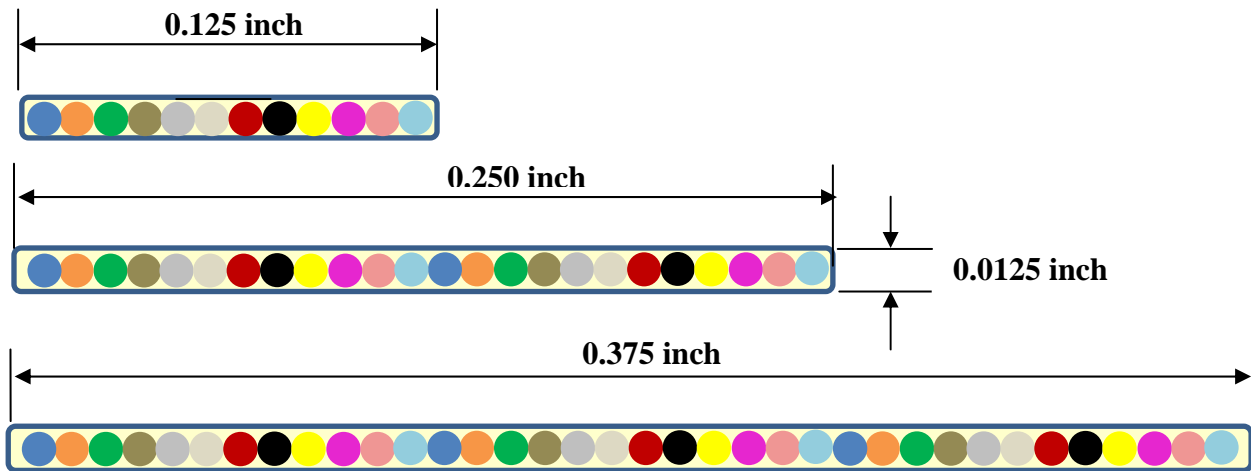


Figure 1: Representative 12, 24, & 36 fiber ribbons with dimensions

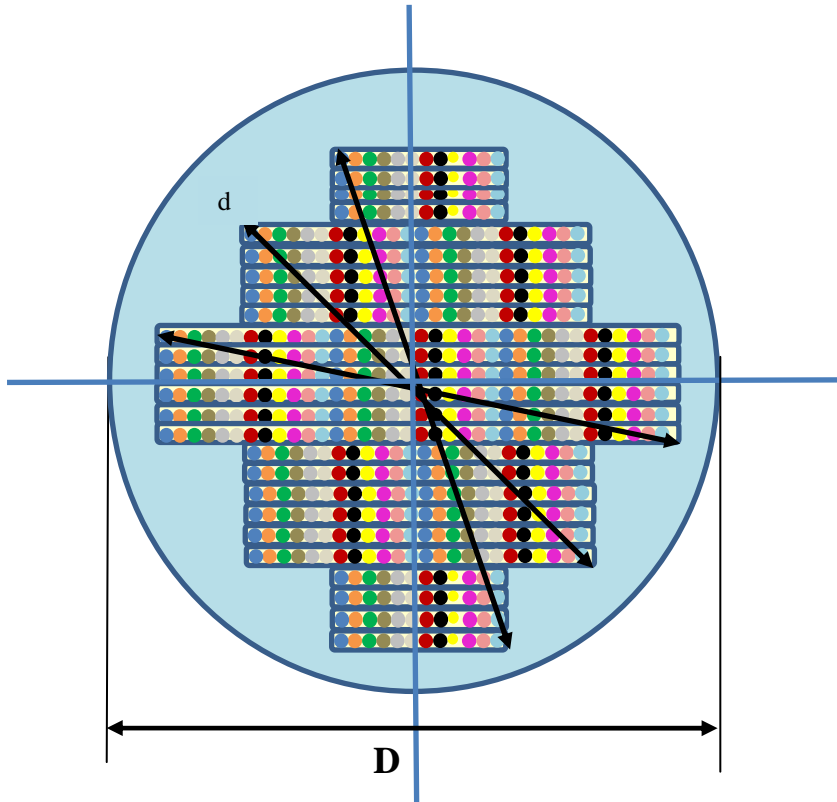


Figure 2a: Exemplary ribbon stack in cable

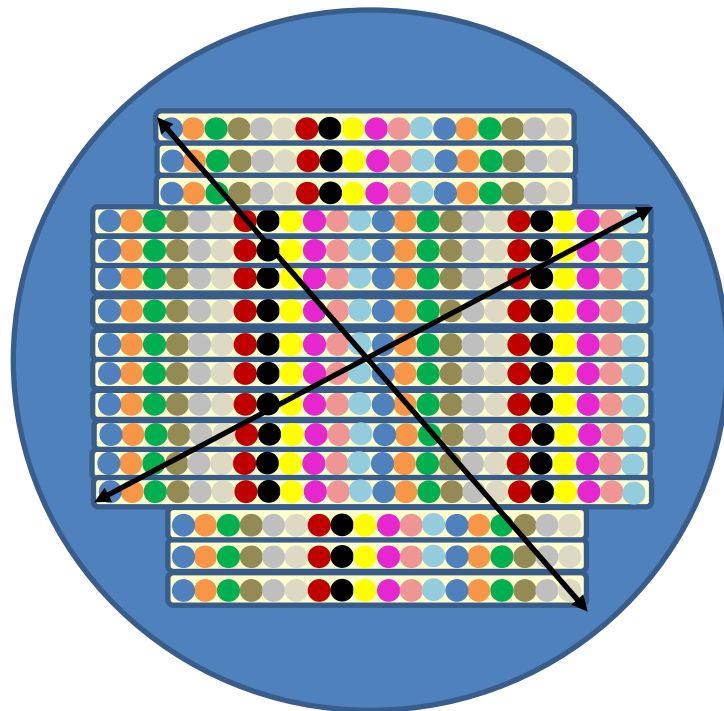


Figure 2b: Exemplary ribbon stack in cable core tube

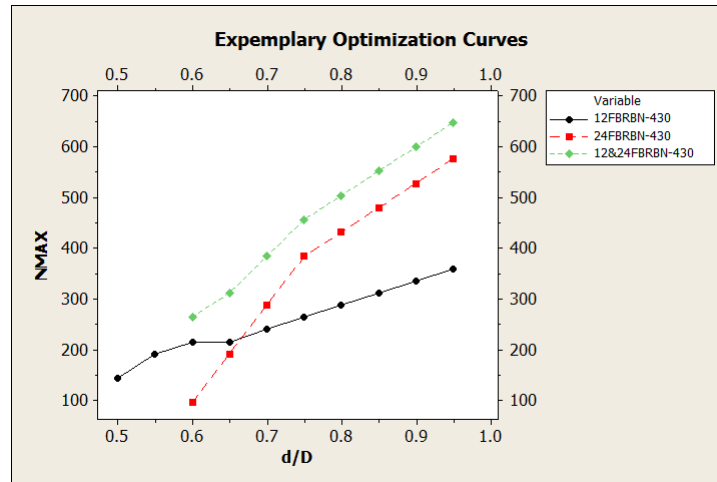


Figure 3: Exemplary optimization curves

Bibliography

1. Ackoff, Russell L. and Daniel Greenberg, *Turning Learning Right Side Up: Putting Education Back on Track*. Prentice Hall, 2008.
2. Ackoff, Russell L., "Toward an Idealized University". *Management Science*, Vol.15, No. 4, December 1968, pp.121-131.
3. Birgin, Ernesto G. Rafael D. Lobato. "Orthogonal packing of identical rectangles within isotropic convex regions". *Computers & Industrial Engineering*, Vol. 59, No. 4, pp.595-602, November, 2010.
4. National Academy of Engineering, "Educating the Engineer of 2020: Adapting Engineering Education to the New Century". Washington D.C.: National Academies Press, 2005.
5. Jackson, K.W. Hardwick, N.E. Kinard, M.D. Santana, M.R. Travieso, R. "Optimal design strategies for central tube ribbon cables comprising 864 fibers and beyond". *International Wire and Cable Symposium Proceedings* 118-126. 1999.
6. Jackson, Kenneth W. and Reichert, Nancy L. "Teaching Engineers to Compete in the 21st Century-A Multidisciplinary Approach for Honors Students". *ASEE SE Section Conference*, 2009.
7. Cochran, J. J. "Introductory Business OR Cases: Successful Use of Cases in Introductory Undergraduate Business College Operational Research Courses" *The Journal of the Operational Research Society* , Vol. 51, No. 12, Teaching OR: A Review of Current Practice and Developments (Dec., 2000), pp. 1378-1385. Published by: Palgrave Macmillan Journals on behalf of the Operational Research Society. Article Stable URL: <http://www.jstor.org/stable/254035>
8. Jackson, Kenneth W. United States Patent No. US 6,487,348 B1, November 26, 2002.
9. Jonassen, David H. "Instructional Design Models for Well-Structured and Ill-Structured Problem Solving Learning Outcomes". *Educational Technology Research and Development*. Volume 45, No.1, pp.65-94, 1997.