AC 2008-1261: USING ROBOTICS TO TEACH MATHEMATICS: ANALYSIS OF A CURRICULUM DESIGNED AND IMPLEMENTED

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Using Robotics to Teach Mathematics: Analysis of a Curriculum Designed and Implemented

Abstract

We report on a project that investigates the use of engineering as a context in which to learn mathematics through an evaluation of a LEGO-based robotics curriculum. We performed a content analysis of the curriculum in order to identify the types of mathematics topics that students would have an opportunity to learn, and investigated the extent to which those topics were aligned with national mathematics standards. The curriculum had a large percentage of tasks with clear relevance for mathematics and aligned well with the standards at the level of broad, topic areas (e.g., measurement, algebra, etc.). The curriculum was not well aligned at the more specific, topic level (e.g., use of measuring instruments, evaluating expressions, etc.), indicating that level of alignment is an important consideration when designing engineering curricula to teach mathematics. We simultaneously conducted a case study analysis of an implementation of the robotics curriculum in an eighth grade technology classroom to assess whether mathematics ideas were salient as students engaged with the tasks. When prompted by the teacher, especially during whole-class discussion, we observed students bringing in a wide range of formal mathematics ideas. Despite that, because of the multitude and diversity of those mathematics ideas, significant mathematics learning did not occur. These findings suggest that robotics is a promising engineering context in which to engage students in thinking about mathematics, but that further supports are required to effectively enable students' mastery of the more general mathematical ideas.

Introduction

Mathematics curricula that is "a mile wide and an inch deep"[1] is often cited as a reason for poor K-12 mathematics achievement in the United States, since it most likely contributes to superficial coverage of ideas, students' views of mathematics as simply a large collection of rote procedures[2], and acquisition of inert knowledge that is learned without consideration for its use outside of the classroom[3]. Engineering design has been proposed as an alternative approach to teaching mathematics (in addition to other disciplines such as science and technology) as it has the potential to serve as an integrator[4], providing a context in which students can synthesize and apply mathematics knowledge in authentic problem-solving situations. Authentic problem solving is not tightly bound by traditional disciplinary or conceptual boundaries, such that many different types of knowledge need to be applied flexibly in order to be successful. This experience of engineering a design solution may, in turn, reinforce students' knowledge of the general mathematics idea. In addition, because engineering design projects are often about satisfying human needs and finding solutions to human problems, they are inherently motivating for students.

Despite these theoretical arguments, the empirical evidence supporting the utilization of engineering design specifically to facilitate learning of mathematics has remained largely anecdotal. Systematic studies of engineering design curricula intended to teach traditional mathematics disciplinary knowledge are not common, although some do exist. For instance, Burghardt and Krowles described a project in which fifth grade students learned geometry concepts by designing a chair.[5] They incorporated short, focused activities called Knowledge and

Skill Builders (KSBs) that provided students' with key mathematics ideas, which then informed the design solution. As new mathematics ideas were introduced, students were required to demonstrate their understanding first, and only then were encouraged to modify their design sketch to incorporate that idea. These pedagogical moves in the enactment of the curriculum helped to ensure that students attended to the mathematics content and not just to their designs.

Robotics, in particular, is an engineering context that is highly engaging for K-12 students, and therefore is a good case for considering the potential synergy of engineering curricula and mathematics learning. In popular student robot competitions, "Making the robot do what I want" is often cited as a motivating factor for participating students to learn general ideas related to engineering and programming.[6] Other research in robot competition settings has shown that there are many educational benefits to students participating in those competitions, including the acquisition of skills related to mechanical engineering and electronics.[7] Mentors of the student teams are likely (80%) to say that their students' math skills were helped through participation in the competition. Interestingly, the students are much less likely (~30%) to report that they learned math skills from their participation, although that may be because they are less aware of their learning than the mentors. Although these results with robotics competitions are encouraging, only a small subset of the K-12 population voluntarily chooses to participate in these competitions and therefore those students who do participate may be especially motivated to learn from their experiences. As a result, the students may not be representative of the larger school population and may hide some of the challenges of implementing robotics curricula in a classroom setting. In addition, the educational benefits are assessed through self-report, so the specifics of what mathematics students learned and to what extent are not known precisely.

Other research has specifically investigated using robotics as an engineering context in which to teach mathematics in the classroom.[8,9] Norton reported a project in which students learned about ratios by constructing cars that utilized gears and pulleys in the drive system.[8] Although on average the students in the class improved on a knowledge test, there were some students who didn't make improvements. The teachers of the class noted that more explicit connections needed to be made between the construction activities and the mathematical ideas so that all students could abstract the general concepts. A second iteration of the curriculum was carried out in a subsequent study with the revisions specifically targeting girls.[9] In particular, to help the students abstract the general ideas related to ratio, two additional activities in distinct contexts were presented to the students prior to the robotics construction activity: a dilution activity and an investigation into whether the anatomical proportions of a Barbie doll are realistic. As a result of these additional contexts in which to explore the mathematics, the students reported less frustration in making the connections between the mathematics and the design task and were also more engaged and interested. This research points to some of the challenges in facilitating students' conceptual development of general mathematics ideas from concrete design activities.

One important factor that may explain much of the success of engineering curricula in helping students to learn mathematics is the construct of opportunity to learn (OTL). There are different dimensions to OTL, but a central aspect refers simply to the content coverage and content emphasis of instruction.[10] OTL based on content coverage is consistently associated with student performance, even after controlling for other common individual and teacher factors.[11] As a result, systematic analyses of the content of a curriculum are essential to understand the possibilities for learning.

For this study, we will analyze a robotics curriculum to investigate the mathematics ideas that students can and do learn from it. The research questions that we will attempt to answer include: (1) In what ways and to what extent is the mathematics content integrated into the *design* of the robotics curriculum? (2) In what ways and to what extent is the mathematics content integrated into the *implementation* of the robotics curriculum? In sum, we are reporting on a project that investigates the potential synergy between robotics engineering and mathematics learning.

Methods

The authors of this paper, whose background is in learning science research, were recruited by the developers of a robotics curriculum to be collaborators in an evaluation of the curriculum in terms of its impact on mathematics learning. Our efforts for improvement involved systematically considering both the design of the curriculum and its implementation in classrooms. The National Research Council report on evaluating the effectiveness of K-12 mathematics curriculum[12] recommended an evaluation framework centered around three methodologies: content analyses, cases studies, and comparative studies. Following this framework, we conducted both a content analysis and a case study analysis of the robotics curriculum. Future work will include comparative studies, so that all three recommended methodologies from the evaluation framework will be taken together to provide a more complete understanding of the effects of the curriculum.

The curriculum that was analyzed for this study was designed for middle school students with the explicit purpose of addressing technological literacy and mathematical competency using robotics as the organizer. Because our focus for this study was on students' learning of mathematics, we will concentrate our efforts here specifically on the mathematics outcomes. The robotics curriculum consisted of a set of multimedia instructional screens viewed in a web browser (see Figure 1 for an example) and print worksheets that led students alternately through six behavior programming modules (e.g., program the robot to move forward, turn, and use its sensors, etc.) and six investigations into mathematics ideas (e.g., the relationship of distance traveled to wheel size and number of wheel rotations). All of the curriculum materials were specifically designed to utilize the LEGO Mindstorms NXT platform and programming software.

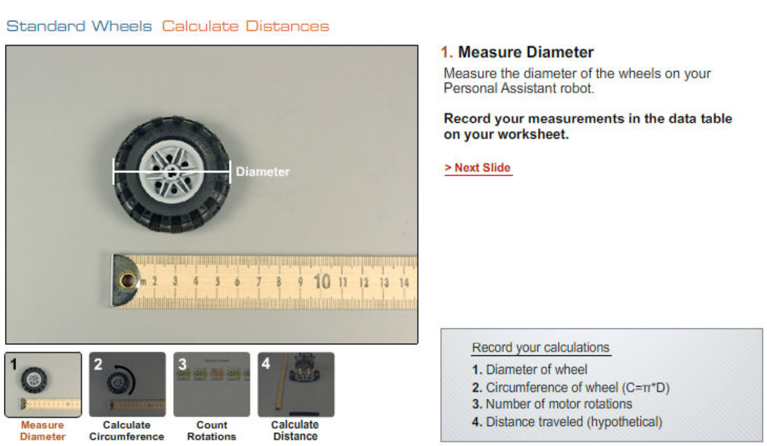

Figure 1: Example Robotics Curriculum Task

Our analysis of the curriculum design utilized the *Surveys of Enacted Curricula* (SEC)[10], a technique to systematically analyze the content of instructional materials, which facilitates the alignment between these materials and accountability documents such as standards and assessments. The SEC tools are based on a set of mathematics topics (e.g., use of measuring instruments, unit conversions) that are grouped into broad topic areas (e.g., Measurement, Algebra, etc.). An eighth grade mathematics teacher familiar with the robotics curriculum coded by hand each task of the robotics curriculum by identifying the mathematics knowledge that was central to completing the task and matching that with the SEC mathematics topics list. Up to three mathematics topics could be assigned to each task. If there were no relevant mathematics topics, then the task was assigned a code of "Not Math". In the rare cases that there was a mathematics topic covered by a curriculum task, but that topic was not present in the SEC topic list, then a new topic was created. There were 215 distinct mathematics topics organized into 17 broad topic areas. We were then able to determine the amount of time devoted to each mathematics topic by calculating the proportion of curriculum tasks devoted to that topic. Similarly, we coded the NCTM's *Standards for School Mathematics*[13] for Grades 6-8 according to the same process. Finally, we calculated a Pearson's correlation coefficient between the topic coding for the robotics curriculum tasks and the topic coding for the national mathematics standards. This allowed us to evaluate the extent to which the content taught by the robotics curriculum aligned with the content that should be emphasized at those grade levels based on the national standards.

In parallel to our content analysis of the designed curriculum, we also conducted a case study analysis of the curriculum being enacted in a classroom. Our analysis of the curriculum in-action was based on our observation of a pilot implementation taught by a knowledgeable instructor in a challenging eighth grade setting. In order to test the efficacy of the curriculum for a broad student population, the setting for the pilot implementation was a school in an urban, public middle school with a high-needs student population (99% minority, 94% economically disadvantaged). The school has a technology and pre-engineering emphasis and the specific class that was observed was a technology class that meets one day a week for 90 minutes. Both a technology teacher and a mathematics teacher normally teach the class by dividing the students into two sections, such that for the first 45 minutes half the students engage in technology-specific activities with the technology teacher and the other half work with the mathematics teacher on mathematics remediation. They then switch for the second 45 minutes. As a result of this dual emphasis on technology and mathematics, there was an explicit goal to teach mathematics in this classroom and connect the mathematics to technology. For this pilot implementation, the class was taught all together for the full 90 minutes using the robotics curriculum over the course of nine sessions. In order to ensure implementation of the robotics curriculum as it was intended, one of the developers of the curriculum materials served as the primary instructor for the class, with supplemental support provided by the two normal instructors. Students worked in pairs or as individuals on the robotics tasks from the curriculum, and whole-class discussion was utilized regularly to encourage students to share their solutions and to provide a forum for consideration of the relevant mathematical ideas. One of the co-authors observed every day and took field notes. Much of the results from the case study are based on these field notes. The researcher did not provide direct instructional support to students, but the researcher, the curriculum developer and the normal classroom teachers did regularly engage in debriefing sessions after each class. Those sessions were an opportunity for the instructors to get feedback and modify classroom practice based on observations of student successes and difficulties.

In addition, a paper-and-pencil pre- and post-test was administered to assess knowledge gains in mathematics. The test included multiple-choice items modified from released items on the state standardized test. Based on an initial, cursory content analysis of the curriculum, we determined that two of the state standards most closely aligned with the curriculum: Rate, ratios, and proportions, and Unit conversions. Eight original items were chosen, such that half the items were designed to assess the rate, ratios, and proportions standard and the other half were designed to assess the unit conversions standard. From the eight original items, eight additional items were written that were isomorphic to the original ones (i.e., contained the same underlying mathematical structure and numerical values) but were modified to include a cover story of a robotics context similar to what students would encounter in the robotics curriculum. The isomorphic items provided us with the capability of detecting whether students were learning mathematics in the context of robotics, but were not generalizing that mathematics to other contexts. An example item is provided in Figure 2. Two forms of the test were constructed, each with eight items, half of which were in their original form and the other half in their isomorphic robotics-context form. Students were randomly given one form for the pre-test and the other form at the post-test in order to eliminate possible effects of test familiarity. The test was kept purposefully short as this was a pilot implementation and our main concern was in assessing qualitatively the extent to which the curriculum tasks encouraged students to think carefully about the mathematics.

Original Context	*Robotics Context*
Mary worked 8 hours and was paid a total of $44. At this rate, how long would it take her to earn $110? a) 60.5 hours b) 20 hours * c) 13.75 hours d) 2 1/2 hours e) 8 1/4 hours	A robot's wheels rotated 8 times and the robot traveled a total distance of 44 cm. At this rate, how many rotations would it take the robot to travel 110 cm? a) 8 1/4 rotations b) 2 1/2 rotations c) 13.75 rotations d) 20 rotations * e) 60.5 rotations

Figure 2: Example Pre-/Post-Test Item (Original and Isomorphic Version)

Although gender differences were not the focus of this study, we recognize the importance of designing STEM instruction that is sensitive to these differences[14], especially to the extent that short-term success in an educational robotics program can lead to long-term career interest in STEM fields. As a result of this class being a requirement for students in remedial mathematics, no student self-selected into the class. Since greater than 48% of the school enrollment is female, it is also unlikely that students self-selected into the school, despite its emphasis on technology and engineering. Nevertheless, recent research with girls has provided some guidance about how instruction with robotics may contribute to maintaining a positive self-image and interest in STEM activities. In particular, positive perceptions about the study of mathematics within a design-oriented classroom can be effectively fostered when sufficient scaffolding is provided to help students understand the connections between design activities and relevant mathematical ideas.[9] We made every effort to make those connections clear whenever possible.

Results and Discussion

Curriculum Design

Our first task was to assess the extent to which the tasks that students are asked to do in the robotics curriculum relate to mathematics at all. Based on the coding, 86% of the tasks from the robotics curriculum had direct relevance to at least one mathematics topic. This indicates that considerable mathematics knowledge is necessary for completing the tasks in the robotics curriculum, and supports the conclusion that the curriculum was well designed with the intention of making the mathematics relevant. For instance, Figure 3 displays the many, different mathematics topics that are relevant in just the first investigation in the curriculum.

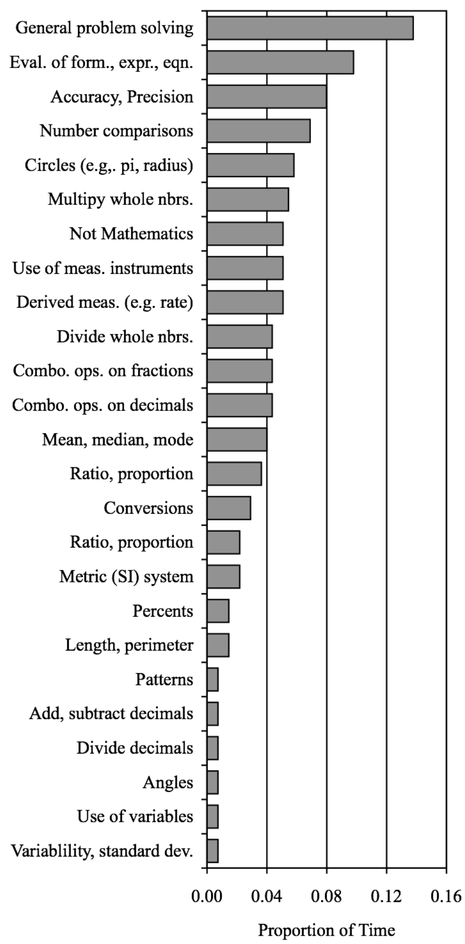

Figure 3: Mathematics Topics Relevant to Investigation 1 of the Robotics Curriculum

Further, we can investigate what mathematics content is being taught in order to assess whether the mathematics that is central to the robotics aligns well with the mathematics content that is supposed to be taught at the eighth grade level. To investigate this, we considered the alignment of the NCTM Standards for Grades 6-8 with the robotics curriculum tasks. Figure 4 illustrates the relative emphasis within the different broad topic areas in mathematics for both the robotics curriculum tasks and for the national mathematics standards. The robotics curriculum aligned well with the NCTM standards (r = .50), indicating that at this level the curriculum is covered many of the same areas of mathematics that are supposed to be covered at this grade level, with the greatest common emphasis being on measurement (27%). It is worth noting that although an alignment score closer to 1 indicates greater alignment, a perfect alignment may not be possible or desirable. In particular, because the robotics curriculum was only intended to last six weeks or less, it is reasonable that it covers some, but not all of the content that is supposed to covered at the target grade levels.

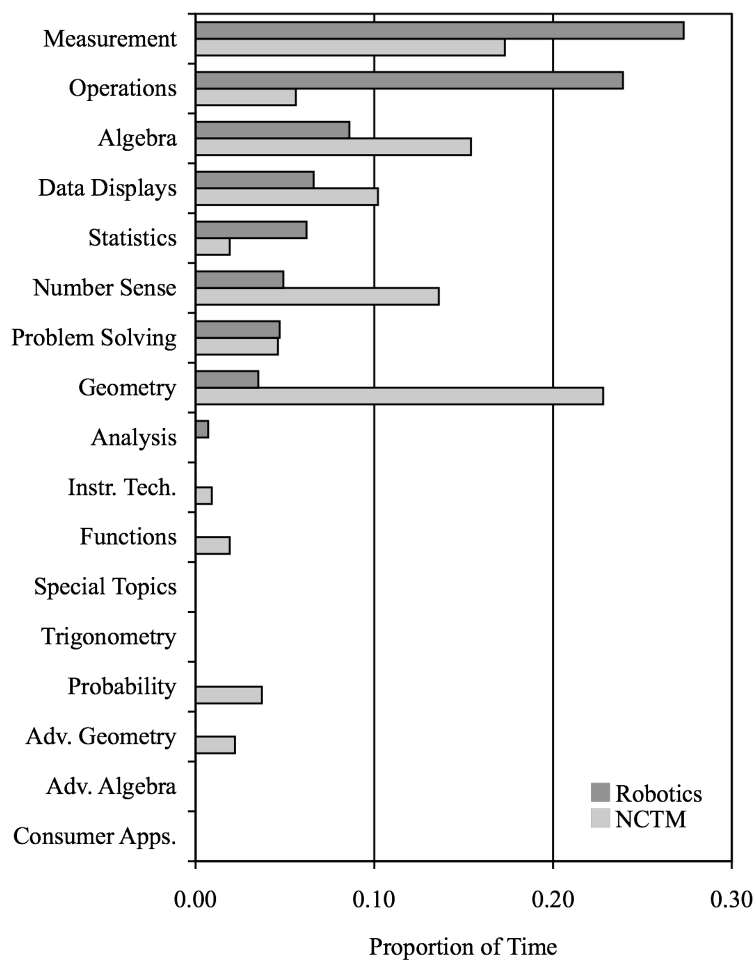

Figure 4: Relative Emphasis of Broad Topic Areas in Mathematics (r = .50)

With the flexibility of the coding scheme and alignment procedure, we were also able to investigate the alignment between the curriculum and the standards at the more specific level of individual mathematics topics. At this level, we found that the curriculum and the standards were not as well aligned (r = -.06). Figure 5 illustrates this misalignment by looking at only the topics within the broad topic area of measurement. In this case, the curriculum tasks emphasize concepts related to the use of measuring instruments and circles foremost, but the standards place more emphasis on measurement concepts related to length, perimeter, area and volume. This indicates that although both the curriculum and standards are emphasizing the same mathematical areas in general, at the more specific level they are each emphasizing different mathematical ideas.

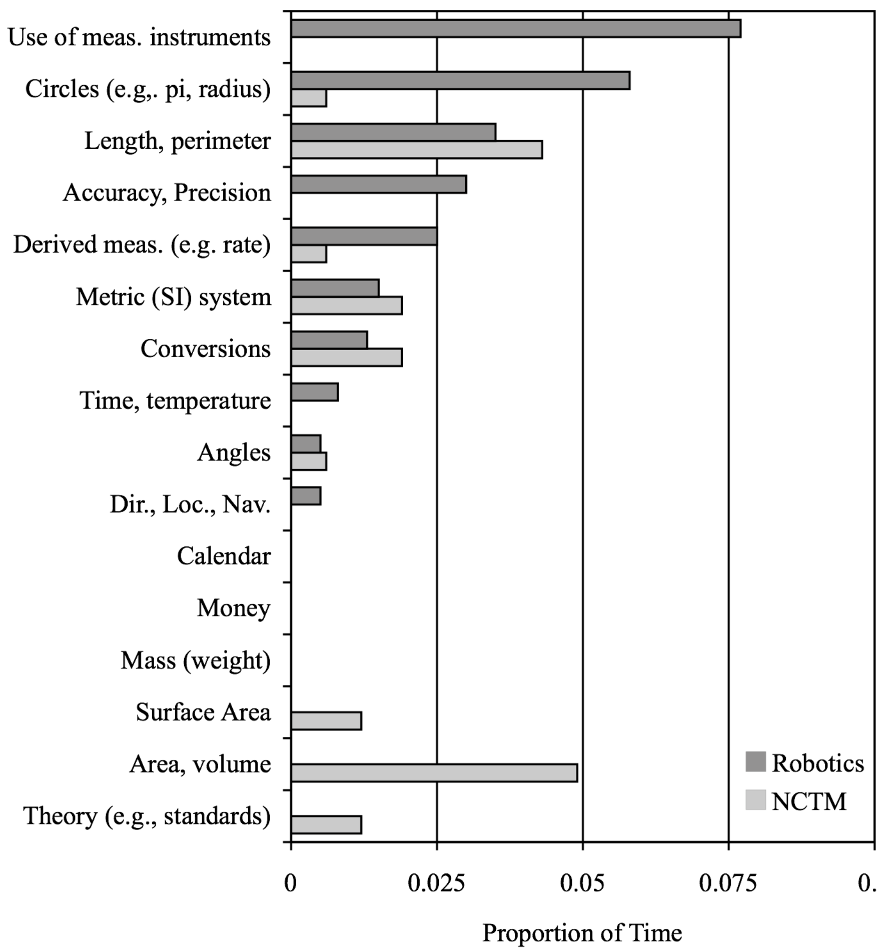

Figure 5: Relative Emphasis of Mathematics Topics within Measurement (r = -.06)

Overall, this analysis suggests that the robotics curriculum was well-designed to cover mathematics content in general, but that the particular content covered may be only loosely-connected to the concepts that are intended to be covered. A further implication of this finding is that a robotics curriculum that is not aligned with standards at the concept level may be less likely to have a measurable effect on mathematics learning using standards-based measures, effectively underestimating its overall impact.

Curriculum In-Action – Mathematics Knowledge Gains

We also investigated the extent to which the mathematics content aligned with the robotics curriculum is conveyed in an implementation of the curriculum by observing it in a real classroom. Overall, 20 students participated in the curriculum pilot implementation, 5 of whom were female. Frequent absences meant that very few students attended every session, and both a pre- and post-assessment were obtained from only 14 of the students. Most students worked in pairs with one robot and one computer terminal per pair, although a few students chose to work as individuals. Table 1 summarizes the results of the pre- and post-test. The results indicate that students did not improve their mathematics knowledge as measured by the test, effectively performing at chance prior to instruction and after instruction. In addition, students did no better with the robotics-context isomorphs as compared to the original items, so the lack of improvement is not due to an issue of transfer.

	N	Mean Score	SD
Pre-test	14	.21	.18
Post-test	14	.22	.11

Table 1: Pre and Post Test Results for Mathematics

Curriculum In-Action – Case Study of Classroom Discussion

Although the test results were disappointing, this was a pilot implementation and our primary focus was on improvement of the curriculum and informing our understanding of how educational robotics may effectively incorporate mathematics learning. As a result, we investigated further into students' interactions with the mathematics while engaging in the robotics curriculum tasks by analyzing our classroom observations. One possible explanation for the lack of observed mathematics learning is that students got so involved with the building and programming of their robots that they didn't devote time and thought to consideration of the mathematical concepts. Our observations of the classroom whole-class discussions suggest that this is not the case. On the contrary, our observations lead us to conclude that so many different topics were raised that it was most likely difficult for students to learn any one of them well. Our content analysis provides converging evidence to support this conclusion. For example, more than twenty different mathematics topics are covered in just the first investigation (recall Figure 3). It was an open question how each of those mathematics ideas would receive attention in a classroom implementation and to what extent they would be integrated with each other.

Analyzing one example class discussion may illustrate how students were challenged by the teacher to consider many different mathematical ideas in the course of investigating the behavioral properties of their robots. The discussion we chose as being representative of typical classroom discussions was one that occurred early in the curriculum as students were beginning to investigate how the distance that their robot traveled forward in a straight line was related to the number of rotations of their wheels. On the day prior to this discussion, students had all programmed their robot to go one meter (100 cm) straight ahead using the standard wheels. Table 2 reconstructs the classroom discussion based on observation notes.

	Observation Notes from Class Discussion	Relevant Mathematics Topics
1.	Instructor introduces the day's activity by summarizing the results of the previous day when students programmed the robot to go 100 cm. *Instructor: "Every robot was a little different, but around 2000 [degrees of motor rotation]."*	Variability
2.	Instructor asks students to program their robot to go half previous distance (now 50 cm), so that students begin to see a pattern.	Patterns, Degrees, Proportions
3.	Students complete task easily, although many students adjust their degrees of motor rotation by small amounts (1 or 2 degrees) to get it "just right".	Accuracy, Precision
4.	Instructor brings students back together as a whole group and chooses a recorder to write down everyone's results on the board at the front of the class.	Data Tables
5.	Student recorder writes the label of the second column to be "Rotations", but as first group presents their findings, the class quickly realizes that "Degrees" would be a more appropriate unit since that is what they programmed into their robots. The recorder is asked to make that change and does so.	Measurement Units, Circles, Degrees
6.	Student groups all present their data. (Table 3 is a reproduction of the final student data that was written on the board.) There is variability in students' results.	Variability
7.	*Instructor: "We need to work with one number, not five. Anyone know a fair way to combine them?"*	Average
8.	Students propose a number of plausible solutions, some valid and others not. *Student 1: "Just use mine" (indicating that the choice could be made arbitrarily)*	
9.	Student 2 proposes aligning the wheels better, indicating a belief that being more careful in running the robots would result in the same value for every robot.	Experimental Error
10.	*Student 3: "The median… the middle number."* Instructor praises this student for the suggestion.	Average, Median
11.	Instructor proposes an alternate form of average, the mean, instead. He then justifies his choice to the class. *Instructor: "We need a fair number for what the average robot will do."*	Average, Mean

Observation Notes from Class Discussion	Relevant Mathematics Topics
12. Instructor walks through the calculation of the mean for each of the two distances, including having the students show their work on the board using multicolumn addition and long division. The values they obtain are 2024 degrees for the 100-cm distance and 1001 degrees for the 50-cm distance.	Mean, Operations on Whole Numbers
13. Instructor compares the two average values. *Instructor: "Would you say that is half?"*	Number Comparisons
14. Instructor provides further support by dividing 2024 in half to get 1012, subtracting 1001 from 1012 to get 11. *Instructor: "How far apart are these two numbers here? Is 11 big compared to 1012?"* This encourages students to think about quantitative ways to test their hypotheses about whether two numbers are different or within some margin of error.	Accuracy, Precision
15. Instructor shows students how to calculate percent error and explains that the numbers are not far off from each other.	Accuracy, Precision, Percent Error
16. Instructor helps students reconnect to the original testable idea. *Instructor: "If you go half as much, can you reasonably expect to go half as far?"* Students agree that they would expect that.	Patterns, Proportions
17. Instructor challenges the students to extend the pattern. *Instructor: "There's obviously a pattern. What would it take to go twice as far? Put into your robot twice that and we'll see how far it goes."*	Patterns, Extrapolation
18. Students calculate the number of degrees of wheel rotation by doubling 2024, creating a new track two meters in length to test their robots, and then testing this value on one of the robots in the class. The robot travels forward about two meters in the testing, as predicted.	Operations on Whole Numbers, Degrees
19. Instructor encourages students to generalize the pattern. *Instructor: "You found half [of 1 meter], you found double, what is 3/4?"*	Patterns, Interpolation

Table 2: Reconstruction of a Class Discussion from Observation Notes

The whole-class discussion on this day included many mathematics topics, all for the purpose of investigating the robot behavior and trying to determine if there was a proportional relationship between the degrees of wheel rotation and the distance traveled. In general, we observed that there were many opportunities in the course of the sessions for students to consider mathematics ideas, and the instructor supported students' thinking about those ideas. For instance, in this discussion the instructor challenged the students to apply high-level mathematics ideas during their problem-solving activities. He guided them in drawing inferences from the range of data that they each collected (see notes 7 and 11 in Table 2) and generalizing a pattern that they could use to program their robots for other distances that they had not yet explored directly (see notes 17 and 19 in Table 2). In addition, students often brought in ideas from their formal mathematics classes (e.g., the median, see note 10 in Table 2), indicating they recognized the usefulness of the mathematical ideas in the current problem-solving context. Despite that, because of the multitude and diversity of mathematics ideas, and students' lack of fluency with those ideas, it seems that these discussions, as they were implemented in this case study, were not sufficient to facilitate students' learning of the general mathematics concepts.

Distance	Degrees
50 cm	1000
100 cm	2018
100 cm	2050
50 cm	1000
100 cm	2004
50 cm	1002
50 cm	1005
100 cm	1025

Table 3: Student Data Shared in Whole-Class Discussion

General Discussion

In summary, the robotics curriculum that we analyzed included a considerable amount of mathematics content, both in its design and in its implementation. The types of topics covered, although aligned well with national standards at the broad, topic-area level, are not aligned well at the specific, topic level, highlighting the importance of level in conducting content analyses. In addition, even in a very challenging school environment, the enacted curriculum did involve a significant amount of mathematical thinking, as we observed in a case study of a typical class discussion. Despite this, students in this context did not learn the general mathematics ideas as a result of participating in the pilot implementation of the curriculum.

There are a number of implications of this research. First, opportunity to learn, as assessed through detailed content analyses like the one conducted here, is likely to be a better predictor of learning than binary checklists that are the typical practice in alignments of curricula to standards. Second, the level at which the alignment occurs is of crucial importance, such that a high alignment at the coarse grain size does not guarantee a high alignment at the fine grain sizes. The finer grain sizes may be much more informative about the kinds of ideas students will consider in their talk, which are the kinds of ideas that will be useful in their problem solving. And third, when looking from the perspective of a fine-grained analysis of lesson content, we now know that a single robotics lesson can easily involve a very large number of different mathematics constructs. This is likely to be overwhelming to students who have not yet mastered the majority of these constructs, rather than serving as an integrator of those concepts. In general, applied, rich problems, found in most engineering contexts, are likely to have a similar propensity to involve a diverse set of mathematics.

As a result of this study, we have a number of possible ideas that will help guide our future research with this curriculum and with other engineering curricula intending to teach mathematics. For instance, it may be more appropriate to use the engineering design activity as a capstone activity after the prerequisite mathematics knowledge has already been learned in a more traditional way. The engineering activity might then serve to strengthen and reinforce understanding of the mathematics ideas rather than being the primary way to learn them initially. A second possibility would be to simplify the design tasks in order to reduce the number of math concepts that are relevant for a given lesson. This would allow for focusing on a manageable set of mathematics ideas at any one time. A third possibility might be to have the mathematics ideas be applied across many different contexts, engineering contexts or otherwise. Making the general mathematics ideas explicit in these different contexts would help students to abstract the general features and feel confident in applying the knowledge more flexibly.

Take together, these findings suggest that robotics is a promising engineering context in which to engage students in thinking about mathematics ideas, but that integrating many different mathematics ideas in one concrete context is challenging. Further supports, such as using the design context as a capstone activity or incorporating structured transfer activities, may be required to effectively enable students' mastery of the more general mathematical ideas. In our future work, we intend to consider more case studies of the curriculum in-action with improvements to the implementation based on these findings, in addition to conducting comparative analyses of the curriculum relative to alternative approaches. This will help us to further elaborate on the conditions necessary for designing effective K-12 engineering curricula.

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