



Using Teacher Feedback to Improve the Design of a Fourth Year High School Mathematics Curriculum

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Using Teacher Feedback to Improve the Design of a Fourth Year High School Mathematics Curriculum (Research to Practice) Strand: K-12 Engineering Resources: Best Practices in Curriculum Design

Abstract

This paper presents an evaluation of a high school mathematics curriculum, NICERC's Advanced Math for Engineering and Science (AMES), through high school teacher feedback along with conclusions from this evaluation. First, the reason behind creating such a curriculum is discussed, followed by a description of the curriculum as well as the implementation process, and lastly the evaluation and conclusion sections.

The motivation for creating AMES evolved from a variety of reasons. Initial discussion and research led the developers to believe that a need for high school students to be fluent in specific mathematic concepts directly connected to engineering and science existed. Beyond the desire to improve student's grasp of the material, the new Common Core State Standards (CCSS) necessitated a curriculum that assisted teachers in executing these standards, primarily mathematical ones but also touching on language arts.

The curriculum itself contains four major threads: Coordinate Systems; Vectors and Matrices; Fundamentals of Mathematics; and Conic Sections. Within each thread, a variety of units are included. For each unit, the outline is as follows: introductory activity/background, core content lessons, and a cumulative activity. Furthermore, pertinent historical facts and dates are incorporated as well as activities that require students to debate, write, or present.

Implementation of this curriculum is still in the early stages as this is the first year for it to be piloted. Therefore, teacher feedback was gathered via two primary methods: surveys and the recording of verbal discussion during working group sessions. Conclusions will be made from the analysis of this data using the qualitative method of ethnography due to the open-ended nature of the surveys and field notes compiled.

Introduction

Why create a new secondary high school mathematics curriculum? Because mathematics is at the core of engineering. However, it is also quite often a stumbling block for many students in their learning. Many students are not prepared for college level classes, particularly in mathematics [1, 2, 3, 4]. Point in case, one study evaluated true college-level freshmen and sophomore students entering into STEM disciplines on their knowledge of high school math because it was noticed that they struggle with basic mathematical concepts that are covered at the high school level. Two major conclusions stemmed from the research: one, students specifically struggle with seven particular high school topics [5], and two, students who take more mathematics classes, whether at the high school or collegiate level, are apt to perform better in math and engineering classes [2, 5]. Expounding upon the primary conclusion, the specific areas of weakness were: interpretation of a graph, equation of a line, volume of a prism, volume of a cylinder, cross product, dot product, and matrix multiplication (2x2s). Students were also tested in their knowledge and understanding of Calculus material, including derivatives and

integrals, but scored significantly higher on those questions. As listed above, the high school concepts were more of a struggle than the more advanced mathematics. In addition to this evidence, other researchers have commented that STEM professors perceive that college students often struggle with basic mathematical concepts found in algebra and trigonometry classes [5]. The secondary conclusion is supported by numerous papers, such as “Freshman-Level Mathematics in Engineering: A Review of the Literature in Engineering Education” [1] that says, “if students had further exposure to mathematics, then their mathematical weaknesses could be eliminated.”

If these conclusions are accurate, then it stands to reason that there is a need for a curriculum to be created that covers, in depth, basic algebraic and advanced mathematical concepts in concert with topics that engineering students and experts think should be emphasized at the high school level for those pursuing a career in a STEM field. In addition, research confirms that offering another mathematics class for high school students can only increase their chances of succeeding in higher education. The result of this thinking led to the creation of a curriculum purposely made for students who intend to continue their education in a STEM discipline, and it is called Advanced Math for Engineering and Science (AMES).

Another motivation for the creation of this high school mathematics curriculum is the new system of standards for K-12 schools, the Common Core. Since the initial implementation of these standards is specifically for K-12 mathematics and language arts, it’s a particularly pertinent consideration. Not only does the course hit multiple mathematics Common Core State Standards (CCSS), but it also branches into language arts standards with its cross disciplinary approach which includes writing assignments, classroom debates, and student presentations. Beyond concrete skills, the curriculum also aims for students to improve upon soft skills and higher order thinking as well as creativity; this aspect of the curriculum meshes well with the standards for mathematical practice that should occur at every grade level. For example, the opening project of the AMES curriculum that introduces the Cartesian coordinate system involves students guiding a peer to a certain spot in the class using only verbal instructions. Unless the student giving directions is very specific, the student following the instructions may end up in a completely different position than the directing student intended. The resulting affect on the students is that they see the need for an agreed upon coordinate system, but also that specificity is an important concept. Therefore, the assignment directly correlates to CCSS.Math.Practice.MP6: Attend to precision [6].

Curriculum Description

Four main threads frame the AMES curriculum: Coordinate Systems; Vectors and Matrices; Fundamentals of Mathematics; and Conic Sections. Within each thread, multiple units are listed; for example, Coordinate Systems includes the Cartesian Coordinate and Polar Coordinate units. At this point in time, the complete list of units is as follows:

- Cartesian Coordinates
- Polar Coordinates

- More on Conics
- Complex Plane
- Three Dimensional Coordinates
- Vectors
- Matrices
- Fundamentals of Mathematics

A unit consists of an introduction, core content, and a final project. The introduction section provides background on the topic, relevant historical connections, and an exploratory activity to spark student engagement and interest. Core content lessons present material in such a way that it is challenging to the student and may require intense higher order thinking, but also leaves the student with a deep understanding of the topic. An additional part of the content is when a concept or example correlates with a CCSS, it is tagged according to whichever standard it covers. The final project is an assignment that forces students to utilize all the skills learned in the corresponding unit, in the context of a practical application of the material covered when applicable.

Overall, the integration of fourth year mathematics CCSS, historical components, writing assignments, classroom debates, vocabulary activities, technology lessons, and engaging hands-on projects along with a variety of refresher topics essential to engineering and science professions provides a holistic learning environment for students. These components coincide with research that shows the method of curriculum implementation is just as important, if not more important, than the base content [7]; hence, a STEM curriculum is made that includes introductory and cumulative projects as well as cross-circular content.

Curriculum Implementation

As the AMES curriculum is in the first-year pilot phase, the implementation of it is still in the early stages. For the 2013-2014 school year, one high school in Louisiana and one in Arkansas will pilot select sections of the content in Advanced Math, Algebra II, and Geometry classes. Because the course is a combination of refresher material and fourth year CCSS, the curriculum in its entirety cannot be contained to one class during this pilot phase. In the future AMES could be taught as a fourth year course or an elective math course. Before the course was incorporated in the classroom, professional development workshops were held so that feedback could be gathered during the creation of the curriculum. Two main teacher workshops were held; the three teachers/administrators whom would be putting AMES into practice in the classroom gathered for one workshop, and a separate workshop was open to any teachers implementing any other NICERC curricula.

The teachers piloting the course this year attended a workshop that met twice over the span of the summer for a period of three days at a time, for a total of six days. During these “working group” sessions, the primary developer acted as if the teachers were actually students and having

them complete various introductory and cumulative projects pulled from the curriculum. Besides activities, teachers also spent time reading core content lessons as well as brainstorming ideas for alternate ways to present material and more hands-on projects.

Another workshop was held as an optional one night session during a summer professional development week, National Integrated Cyber Education Research Center's (NICERC) Education Discovery Forum (EDF), for teachers whom are implementing other NICERC curricula. In general, the attendees were high school mathematics and science teachers. This workshop was a condensed version of the working group sessions except the brainstorming component was not included. For three hours, teachers participated in activities and saw content from three units: Cartesian coordinates, polar coordinates, and vectors. After completing the workshop, teachers were asked to take a survey evaluating the presented material which leads to the feedback portion of the paper.

Curriculum Evaluation

Teachers provide a unique perspective on curricula through their experiences working with high school students and being in the classroom. Therefore, both workshops included a time for the teachers to evaluate AMES. Two methods of evaluation were implemented: surveys and verbal discussion. Surveys were completed by both sets of teachers, but contained slightly different questions. The nine high school teachers who participated in the three hour long session highlighting the AMES activities and content filled out a total of three surveys; one immediately after the session, and two at the end of EDF. The two taken at the end of the week included a variety of questions concerning their overall experience at the EDF as well as questions specific to the AMES curriculum. The survey given during the evening workshop session contained these four questions:

1. What did you like about the session?
2. Anything of which you want to see more or less? (List 3 or more)
3. Name 2 parts that will work best in the class.
4. Name 2 parts that students may struggle with in the classroom.

The piloting teachers answered the same set of four questions for every *unit* of the curriculum created at the time (Cartesian Coordinates, Polar Coordinates, Matrices, and Vectors).

The verbal aspect of the feedback occurred in a less formal setting; the working group had an open discussion of strengths and weaknesses of the course, suggestions for additional content, and grade level appropriateness of the curriculum. Field notes, written observations of the discourse, were compiled during this time by a participant observer. The notes primarily cover a period of three days during the initial working group session, but also contain remarks from the second workshop.

To evaluate this data the qualitative method of ethnography was chosen, for two particular reasons. Before explaining the reasoning, the definition of this method, in a modern sense, is one

“... (which) involves the ethnographer participating, overtly or covertly, in people’s daily lives for a extended period of time, watching what happens, listening to what is said, asking questions- in fact, collecting whatever data are available to throw light on the issues that are the focus of the research” [8]. Therefore, the first reason for choosing ethnography is that the situation lent itself to that type of evaluation as during the period of time when the working group convened, the participant observer spent the whole of the day with the teachers and curriculum developers. During that time, the participant observer watched the happenings, voiced personal opinions, and took extensive notes.

The second reason stems from the stage of implementation. Since the AMES curriculum is in the first phase of piloting, data is not yet available to determine effectiveness with respect to student work and response; a large amount of quantitative information is not available yet. However, experienced teachers can provide a valid viewpoint on the possible success or failure of a curriculum. In that vein, ethnographical field notes are described as, “records of discussions, chance conversations, interviews, overheard remarks, observational notes, the researcher may also employ audio and video recordings and quantitative data gathered from surveys or structured observation” which leads to a secondary reason to choose this method since the data gathered all falls within the scope of field notes [9]. To this extent, the feedback collected from the field notes and open-ended survey questions from the three teachers (the ones prepared to pilot the course) will be chiefly analyzed as well as the closed and open-ended survey responses from the other cohort of teachers.

Results

Analyzing responses from the piloting teachers, the data could be categorized into three main sections: negatives, positives, and responses to feedback. Within the negative commentary, the responses are then split into two more categories: technical and content. All of the feedback and responses that follow are summarized using the field notes and surveys mentioned previously.

On the technical side, a primary concern is the time such a course, like AMES, would take. Including introductory and cumulative projects in each unit adds days of class time that are not usually accounted for in a normal mathematics class. Directly connected to time constraints is a teacher’s ability to adhere to a school’s pacing guide – schedule set by the school board or school for what lessons should be covered at what time. Adding projects to a time guideline that normally only allows for sporadic projects or discovery lessons makes it hard for the teacher to stay on track in view of their supervisors. Again, because the course is different than the traditional class, more time is allotted within the lessons for student exploration, which may cause lessons to spill over into additional class periods.

Another technical issue is that the content included in AMES crosses over into multiple high school classes. For example, “More on Conics” and “Three Dimensional Coordinates” are currently included in some high school geometry classes. So, at this time, the teachers feel that AMES could be expanded to become two full courses: Advanced Math and Geometry.

Looking at the negative comments concerning the content, the teachers who would potentially pilot the course thought that grading the more open-ended activities and projects could be a problem. General answer keys would be included with a teacher edition of AMES, but because projects are not a clean-cut grading assignment, no exact rubric to ensure quick and easy grading is available for such projects. Also, certain assignments within the curriculum require students to write an essay or create arguments based off historical information. Grading these non-STEM activities would be out of the STEM teachers' comfort zone.

On the opposite end of the spectrum, several positive comments were made. One big draw for the teachers is the denotation of which examples and activities connect to CCSS. Figure 1 shows an example problem from AMES that meets CCSS N.VM 10 which states that students should "Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse" [6].

Ex 6: Does the matrix $F = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$ have an inverse? If so, find it.

First, calculate the determinant.

$$\begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = (7 \times 1) - (2 \times 3) = 1$$

The determinant is not zero; therefore, now find the inverse.

$$\begin{bmatrix} 7 & 2 & : & 1 & 0 \\ 3 & 1 & : & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & : & 1 & -2 \\ 3 & 1 & : & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & : & 1 & -2 \\ 0 & 1 & : & -3 & 7 \end{bmatrix}$$

CCSS N.VM 10

Figure 1

Besides the concrete mathematical standards, the introductory and cumulative activities often naturally address the first set of "standards for mathematical practice" (standards that, according to the Common Core State Standards initiative, are more representative of soft skills that should develop along with the other standards in every grade) [6].

The most consistent feedback response is that one of the best features of the curriculum is the engaging content, particularly the activities. Besides simply stating their affinity for the activities, teachers also mentioned certain projects being favorite parts of the workshop. A reason why some of the activities are so engaging is the collaborative nature of the assignments, not only between students but also between teachers. For the students, one example of collaboration is the cumulative project called Urban Design; students team up to create a urban design plan for a section of city using their knowledge of lines, parametric equations, and

parabolas. Beyond the teamwork required to create this plan, one of the piloting teachers even suggested (and the idea was hence implemented into the curriculum) that student teams switch plans and check another team's set of equations for a certain area of their city. From the teacher perspective, when a mathematics teacher gets to the Cartesian Coordinate writing assignment, the English teacher could be the one to actually assign the essay. This collaboration pairs teachers from multiple disciplines while also could provide students with multidiscipline connections.

The use of technology was another favorable element of AMES according to the teachers, particularly the use of a free software program called GeoGebra to assist students in learning. While they enjoyed that technology use was already imbedded within the curriculum, the piloting teachers also asked for more technological connections throughout the lessons.

The rigorousness of the content is also a highlight. For example, five different equations that represent lines are taught in the course, but after completing work for each equation students are tasked with deriving one equation from the other four equations. So, a student could begin with the point-slope formula and have to derive the slope intercept form, two point form, intercept form, and general form of a line, respectively. Figure 2 gives a visual explanation of the problem.

Advanced Math for Engineering and Science	Plane Analytic Geometry	Notes	Slope Intercept Form $y = mx + b$	Point Slope Form $(y - y_1) = m(x - x_1)$	Two Point Form $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	Intercept Form $\frac{x}{a} + \frac{y}{b} = 1$	General Form $Ax + By + C = 0$
			Slope Intercept Form $y = mx + b$	Point Slope Form $(y - y_1) = m(x - x_1)$	Two Point Form $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	Intercept Form $\frac{x}{a} + \frac{y}{b} = 1$	General Form $Ax + By + C = 0$
			Solutions for this column are on page a.	Solutions for this column are on page b.	Solutions for this column are on page c.	Solutions for this column are on page d.	Solutions for this column are on page e.

Figure 2

The graphics and use of color conclude the list of positive feedback categories. Examples intended to be led by teachers are placed in orange boxes, and problems within the lesson that are intended for students to work during class are outlined in blue. While this may not seem like a major draw to casual observers, teachers say that even a little bit of color in a math text makes it more engaging than normal. In addition to this, colored graphics accompany certain problems

when a visual could be helpful in understanding the question, and the feedback expressed that the pictures were a good component.

As the workshop was a working group, one in which changes could be made to the curriculum in reaction to the teachers' suggestions, the developers were able to respond to the feedback given by the teachers.

While the teachers expressed concern about the new curriculum not being contained to one subject (i.e. trigonometry, geometry, algebra, or advanced math), the creators feel that as long as the course prepares students for engineering and math classes at the university level, then the goal is being met. Combined with this point of view, the AMES developers are content with the curriculum being labeled as an elective and not a direct replacement for an Advanced Math class.

The lack of a section devoted especially to trigonometry was another drawback for some of the teachers, but the response to this apprehension was that the trigonometry students need for future classes and careers will naturally occur as other lessons are taught. Besides that, a large majority of trig will now be taught in a Geometry class according to the new CCSS. However, as the CCSS are a recent development, the creators would consider developing a "bridge" section that would include all the trigonometry lessons usually covered in an Advanced Math course until the CCSS have been implemented long enough that students no longer require the bridge.

Two more changes to the curriculum are planned in response to the teacher feedback: technology additions and denotation of more CCSS. While a few technology connections are included in the curriculum, in the future lessons specifically teaching students how to use graphing calculators, excel, and more to complete assignments connected to a unit will be a part of the curriculum. For example, at the end of the matrices unit, a lesson will be included that explains how to find the determinant and inverse, multiple matrices, and add/subtract matrices using graphing calculators and excel.

As for CCSS, AMES currently addresses the connections to the mathematical standards, but in the future ELA standards met by writing assignments or classroom debates will also be laid out for teachers to see.

Conclusion

Mathematics is the basis of engineering, but not all students are adequately prepared to succeed in freshman engineering or mathematics courses. One possible solution to this situation is to prepare students at the high school level via a curriculum specifically created for STEM students. Before implementing such a curriculum, getting advice and perspective from those in the trenches of education (high school teachers) would improve the chances of success not only in putting into practice the curriculum but also in student achievement and teacher acceptance.

While evaluating the curriculum, teachers concluded that the course may take too much time to cover in a school year, the content flows into multiple high school courses, grading activities would be an issue, and that not specifically having a trigonometry section would be a problem. From the opposite point of view, everyone agreed that the engaging projects and activities were a major plus for the curriculum. Positive feedback was also given concerning the CCSS that were tagged in lessons, technology and graphics that were imbedded in the curriculum, and that the course was rigorous. Changes were made reflecting the feedback of the working group, and the course is now being piloted at two schools.

Future work will include gathering data concerning student engagement and achievement. NICERC plans to implement the curriculum as its own course in 2 to 3 schools in the fall while implementing certain parts of AMES into existing courses at select other schools. To prepare this phase of the pilot, teachers' continued feedback will determine if more changes need to be made to the curriculum and if the course is achieving its goal of preparing STEM students for their future.

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