AC 2012-3674: VALIDATING OF THE DIAGNOSTIC CAPABILITIES OF CONCEPT INVENTORIES: PRELIMINARY EVIDENCE FROM THE CONCEPT ASSESSMENT TOOL FOR STATICS (CATS)

Ms. Dana Denick, Purdue University, West Lafayette

Dana Denick is a PhD Student in the School of Engineering Education at Purdue University. Dana holds a BS in Mechanical Engineering from Bucknell University, MA in Physics Education from the University of Virginia and MS in Library and Information Science from Drexel University. Her research interests are difficult concepts in engineering and information literacy for engineering.

Dr. Aidsa I. Santiago-Romn, University of Puerto Rico, Mayaguez Campus

Aidsa I. Santiago-Romn is an Assistant Professor in the Department of Engineering Science and Materials and the Director of the Strategic Engineering Education Development (SEED) Office at the University of Puerto Rico, Mayaguez Campus (UPRM). Dr. Santiago earned a BA (1996) and MS (2000) in Industrial Engineering from UPRM, and Ph.D. (2009) in Engineering Education from Purdue University. Her primary research interest is investigating students’ understanding of difficult concepts in engineering science with underrepresented populations. She also teaches introductory engineering courses such as Problem Solving and Computer Programming, Statics, and Mechanics.

Dr. Ruth A. Streveler, Purdue University, West Lafayette

Ruth A. Streveler is an Assistant Professor in the School of Engineering Education at Purdue University. Before coming to Purdue in 2006 she spent 12 years at Colorado School of Mines, where she was the founding Director of the Center for Engineering Education. Dr. Streveler earned a BA in Biology from Indiana University-Bloomington, MS in Zoology from the Ohio State University, and Ph.D in Educational Psychology from the University of Hawaii at Manoa. Her primary research interests are investigating students’ understanding of difficult concepts in engineering science and helping engineering faculty conduct rigorous research in engineering education.

Ms. Natalie Barrett, Purdue University, West Lafayette

Natalie Barrett is a Mechanical Engineering PhD student at Purdue University and is interested in renewable energy. Natalie received a BSME from Florida State University, a MSME from Georgia Institute of Technology, and a MBA from Indiana University. She has taught at Wentworth Institute of Technology as an Adjunct Professor. She has also worked in industry at Pratt & Whitney for several years and served in roles such as Integrated Product Team Leader and Affordability and Risk Manager for the F135 Engine Program.
Validating the diagnostic capabilities of concept inventories:
Preliminary evidence from the Concept Assessment Tool for Statics (CATS)

Introduction

Assessment, specifically assessment for the development of curricula and evaluation of students’ performance with respect to ABET accreditation requirements, has been an important aspect of engineering education. Therefore, engineering educators need to implement rigorous assessment practices in their courses that are both valid and reliable, in a manner that would allow them to have the necessary evidence to improve students’ learning.1 Engineering concept inventories (CIs) have been developed with the intention to be used by faculty to assess students’ understanding of specific concepts. Unfortunately they have been used primarily to assess the effectiveness of instructional techniques.2 Furthermore, traditional psychometric techniques applied to CIs have shown the validity and reliability of CIs to measure specific engineering concepts, but these techniques have not been able to provide instructors with information that would help them predict students’ performance within those concepts.3 Diagnostic assessment techniques, such as the Fusion Model, provide instructors with information about students’ prior knowledge and misconceptions before beginning a learning activity. They also provide a baseline for understanding how much learning has taken place after the learning activity is completed.4

Previous research studies have applied the Fusion Model to various large-scale instruments, including CIs. Specifically, the Fusion Model was applied to the Concept Assessment Tool for Statics (CATS) through a four-phase process.1 In this study, domain experts identified a set of ten cognitive attributes of which mastery is needed to select a correct answer among distractors (common misconceptions).5 Table 1 presents a list of these cognitive attributes and their descriptions. Results from this quantitative research methodology indicated a high diagnostic capacity of CATS.3 Also, student mastery profiles were generated for each question and cognitive attribute.3 These profiles could be used by instructors to design and implement more targeted instruction, in accordance to each student’s cognitive capability.1 Qualitative research methodologies, such as think-alouds, were identified as necessary to validate experts’ recommendations about students’ point of view and demonstration of expected cognitive attributes.1

This paper focuses on a pilot study establishing an interview protocol that would effectively elicit students’ conceptions regarding five CATS items, corresponding to concepts that were identified as more problematic among engineering students. The research question guiding this study was: How do students’ conceptions and misconceptions align with expected skills and mastery profiles of the Concept Assessment Tool for Statics (CATS)?
Table 1. Cognitive attributes identified for CATS

<table>
<thead>
<tr>
<th>Cognitive Attribute</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>Equivalence</td>
<td>Static equivalence between forces, couples, and combinations.</td>
</tr>
<tr>
<td>2</td>
<td>Newton’s 3rd Law</td>
<td>Forces between two contacting bodies must be equal and opposite.</td>
</tr>
<tr>
<td>5*</td>
<td>Contact Forces</td>
<td>Direction of force between frictionless bodies in point of contact.</td>
</tr>
<tr>
<td>7*</td>
<td>Friction Forces</td>
<td>Implication of equilibrium and Columbus Law of friction force (force must be less than or equal to the coefficient of friction)</td>
</tr>
<tr>
<td>10*</td>
<td>Pin on Slot</td>
<td>Representation of pin on slot.</td>
</tr>
<tr>
<td>11</td>
<td>Roller Support</td>
<td>Representation of roller support (one force perpendicular to the contact surface).</td>
</tr>
<tr>
<td>12</td>
<td>Fixed Support</td>
<td>Representation of Fixed support (two forces in the x-y direction and a moment).</td>
</tr>
<tr>
<td>19</td>
<td>Representation and Tension in Ropes</td>
<td>Identifying forces acting on the corresponding blocks. Identifying forces on the corresponding ropes. Representation of forces as vectors (Pythagoras Theorem).</td>
</tr>
<tr>
<td>20</td>
<td>Representation of Forces</td>
<td>Identifying forces acting on the corresponding blocks.</td>
</tr>
<tr>
<td>21*</td>
<td>Couples &amp; Equilibrium</td>
<td>The moment exerted is the same about any point. Consideration of force and moment balance in equilibrium.</td>
</tr>
</tbody>
</table>

Note: Cognitive attributes addressed by the CATS items selected for this study are designated with *.

Facets of Understanding

This research is guided by recommendations from Minstrell’s work on facets of understanding, in which he argued that understanding how students make meaning of concepts will help teachers to design more targeted instruction. In his work, Minstrell has attempted to understand physics students’ thinking for increased cognition in instruction. He argues that most assessment instruments are scored with a two point rubric (either answered correctly or incorrectly) and this process is not useful in determining the student thinking behind incorrect responses or how to help students correct their thinking. The process followed by Minstrell was to first identify and organize students’ thinking toward problematic situations into individual sorts of thinking — also referred to as facets. The concept of facets was used to describe students’ thinking as it was seen or heard in the classroom and represent individual pieces of students’ knowledge or strategies of reasoning. These facets were then clustered according to certain ideas students expressed. Later for each cluster, facets were organized as: (a) appropriate or acceptable understanding for introductory physics, (b) arising from formal instruction, but either overgeneralized or undergeneralized in application, or (c) more problematic and needing instructional intervention to prevent student difficulty with the cluster or ideas in related clusters. In summary, Minstrell recommends the use of qualitative research strategies, such as open-ended interviews, focus groups, or think-alouds, to diagnose students’ misconceptions.

Fusion Model

The Fusion Model is a statistical technique that models student response behavior by both a binary attribute (mastery or non-mastery) and a continuous attribute (mastery differing by degree). The model is based on a Q-matrix approach in which the Q-matrix is a binary representation of the underlying cognitive attributes required for correct item responses.
Additionally, the Fusion Model uses residual information from a continuous attribute to uniquely determine a student’s probability for correctly performing each task. The Fusion Model employs a Bayesian approach to estimate the model parameters and estimations are made based on a Monte Carlo Markov Chain (MCMC) parameter estimation algorithm. The Fusion Model has shown promising results when applied to real educational assessment data and in the case of CATS, a previous study found indication of a high correspondence between observed and estimated item difficulty, a high accuracy in estimating students’ cognitive attributes mastery probability, and a fit between observed and estimated score distribution. Additionally, the application of the Fusion Model indicated a high diagnostic capacity of CATS and produced expected patterns of cognitive attributes mastery as seen in Table 2.

Table 2: Estimated Cognitive Attribute Profiles

<table>
<thead>
<tr>
<th>Top Five Mastery Patterns by Appearance:</th>
<th>Highest Occurrence of Non-Mastery:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Non-mastering of all cognitive attributes</td>
<td>7 (friction force)</td>
</tr>
<tr>
<td>2. Mastering all cognitive attributes</td>
<td>2 (Newton's 3rd Law)</td>
</tr>
<tr>
<td>3. Mastering only attribute 11 (roller support)</td>
<td>5 (contact forces)</td>
</tr>
<tr>
<td>4. Mastering altogether attributes 10 (pin on slot), 11 (roller support), 12 (fixed support), 19 (representation and tension in ropes) and 20 (representation of forces)</td>
<td>21 (couples &amp; equilibrium)</td>
</tr>
<tr>
<td>5. Mastering attributes 10 (pin on slot) and 11 (roller support)</td>
<td>1 (equivalence)</td>
</tr>
</tbody>
</table>

**Research Design**

**Participants:** The study described in this paper looked at interviews from five undergraduate engineering students who had completed statics within the prior academic year; the sample consisted of two female students and three male students. The students’ grades in the statics courses ranged from A to D, with one student retaking the course at the time of the interview. The study was intended to pilot an interview protocol, and as such, the number of participants was limited to a sample size that would prove manageable under constraints.

**Instrument:** CATS is a multiple-choice instrument designed to diagnose students’ misconceptions of statics concepts. CATS items were designed to detect errors highlighting incorrect concepts. The developer of CATS designed three items for each of the nine concepts tested on the instrument: (a) Drawing forces on separate bodies, (b) Newton’s 3rd Law, (c) Static Equivalence, (d) Roller joint, (e) Pin-in-slot joint, (f) Loads at surfaces with negligible friction, (g) Representing loads at connections, (h) Limits on friction force, and (i) Equilibrium. Also, each problem had been carefully designed to identify conceptual errors or misconceptions, without the need for mathematical computation. Additionally, the developer of CATS, Dr. Steif, has identified a set of distinct errors that reflect known misconceptions exhibited by students based on his experience and occurrence in student documentation. Table 3 presents a list of these errors and their descriptions.
Table 3. Conceptual Errors in Statics

<table>
<thead>
<tr>
<th>Error</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Failure to be clear as to which body is being considered for equilibrium.</td>
</tr>
<tr>
<td>2</td>
<td>Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two.</td>
</tr>
<tr>
<td>3</td>
<td>Leaving a force off the free body diagram (FBD) when it should be acting.</td>
</tr>
<tr>
<td>4</td>
<td>Drawing a force as acting on the body in the FBD, even though that force is exerted by a part which is also included in the FBD.</td>
</tr>
<tr>
<td>5</td>
<td>Drawing a force as acting on the body of the FBD, even though that force does not act directly on the body.</td>
</tr>
<tr>
<td>6</td>
<td>Failing to account for the mutual (equal and opposite) nature of forces between connected bodies that are separated for analysis.</td>
</tr>
<tr>
<td>7</td>
<td>Ignoring a couple that could act between two bodies or falsely presuming its presence.</td>
</tr>
<tr>
<td>8</td>
<td>Not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces.</td>
</tr>
<tr>
<td>9</td>
<td>Presuming a friction force is at the slipping limit (µN), even though equilibrium is maintained with a friction force of lesser magnitude.</td>
</tr>
<tr>
<td>10</td>
<td>Failure to impose balance of forces in all directions and moments about all axes.</td>
</tr>
<tr>
<td>11</td>
<td>Having a couple contribute to a force summation or improperly accounting for a couple in a moment summation.</td>
</tr>
</tbody>
</table>

**Methodology:** Based on previous studies establishing skills and misconceptions as identified by CATS, a set of five items were selected for this pilot study. These items addressed difficult cognitive attributes as determined by the Fusion Model, specifically: Pin on slot, Equivalent Forces, Contact Forces, Friction Force, and Couples & Equilibrium. An interview protocol was developed that took into consideration the characteristics of the CATS questions and associated distractors, and their relationships to the cognitive attributes and common errors presented in Tables 1 and 3. A summary of the cognitive attributes and common errors identified for each item is presented in Figure 1 below.

![Figure 1. Cognitive attributes and statics errors identified for each CATS item](image-url)
design included the question of whether students are applying procedural rules and following analytical steps or if students are applying mental models, perhaps involving spatial visualization, to approach the CATS items. As students’ reasoning and thinking cannot be determined from response behavior alone, the protocol was designed to encourage students to explain what they are thinking by explicitly asking students to explain why their selected response is correct and why other answers are incorrect. A preliminary version of the protocol can be found in the Appendix.

A sample of five undergraduate engineering students was obtained through flyer and email recruitment. The students were required to have completed a statics course within the previous academic year and be able to explain their thought process when solving a problem in fluent English. Students that met eligibility were then interviewed and compensated for their participation. In the interviews, each student was presented with the five selected items in a prescribed order, presenting the items from least difficult to most difficult. Students were encouraged to verbally explain their thinking while solving the selected statics problems. Audio recordings were taken at the time of the interviews and transcripts were created for analysis of students’ demonstrated conceptual understanding.

Four researchers analyzed participants’ transcripts to identify: (1) statics concepts students used to solve the problem, (2) conceptual errors students made and also (3) the presence of the cognitive attributes previously identified by experts for each of the items selected.

Results

The following section presents a description of the cognitive attributes and common errors for each CATS item as identified by domain experts. Additionally, student interview excerpts are provided that validate the presence of these cognitive attributes or conceptual errors. In the case of a new attribute or error arising from the student interviews, the following section includes supporting evidence and explanation.

Item 1: Q-13
Cognitive Attribute Assigned: Pin on Slot (C.A. 10)

13. The mechanisms is acted upon by the downward forces shown. The pin in the slot is frictionless. What is the direction of the force exerted by the slot on the pin A?

Analysis of Student Responses: Three of the five students selected the correct response, B. Of these correct responses, two students indicated both aspects of the concept: the force would act at the point of contact and perpendicular to the slot. The third student identified the perpendicular direction of the force through hand signals, but did not explicitly verbalize this or indicate that the force would act at the contact point.
**Student B:** “the only force will be the normal force by the pin, by the slot... the force will acting towards the left on the pin.”

**Student D:** “the normal force is going to act perpendicular to the-- to where the pin is touching”

**Student A:** “I can kind of tell like which side of the pin that the force may be pushing on the slot and like, or which side of the slot the pin’s force will be pushing on as it’s pushing or as it’s moving and then just trying to make sure the direction will make sense with how I’m thinking.”

Despite arriving at the correct response, Student A’s explanation may indicate that the student does not sufficiently restrict the possible forces as described by Common Error 8. The student leaves open the opportunity for multiple force locations by not including the specification that the force must act at the contact point.

The remaining two students initially chose the correct response B and then changed responses, one to an answer not provided and one to answer E, which shows the normal force in combination with a moment about the pin.

**Student C:** “the normal force exerted by the slot is going to be perpendicular to the plane of the slot... The force that the slot exerts on the pin. It can not exert at the moment about the pin, because there’s no moment arm... I think the answer’s negative B.”

**Student E:** “the normal force is the force that the slot would exert onto the pin. So it would be sideways... I guess it should be E now that I know it’s a moment, because that’s also a reaction force due to the rotational motion of this arm.”

By selecting the negative of the correct response, Student C demonstrates Common Error 3: leaving a force off the free body diagram when it should be acting. In this case, the student does not recognize the pair of equal and opposite forces acting at the contact point. Student E’s error shows a lack of understanding of the characteristics of pin on slot mechanisms and allows for the occurrence of a moment at a point. This evidence suggests that an additional Common Error 12 may be considered as presented in Table 4. The previous explanation of the existence of a moment at the pin also suggests that Student E fails to understand Cognitive Attribute 10, representation of pin on slot. Interestingly, both students mentioned the presence of the spring altering their perception of the problem, however it is unclear if this can be associated with an identifiable error.

<table>
<thead>
<tr>
<th>Experts’ Common Errors</th>
<th>Errors in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two.</td>
<td>2 No identified evidence</td>
</tr>
<tr>
<td>3 Leaving a force off the free body diagram (FBD) when it should be acting.</td>
<td>3 Student identifies force exerted by the pin on the slot, but fails to recognize presence of opposing force.</td>
</tr>
<tr>
<td>8 Not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces.</td>
<td>8 Student does not restrict the location of the force exerted by the slot on the pin to the contact point.</td>
</tr>
<tr>
<td>12* Student allows for presence of a moment at a point.</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Errors generated from student responses identified with *. 
Item 2: Q-7
Cognitive Attributes: Equivalence (C.A. 1), Couples & Equilibrium (C.A. 21)
Common Errors: 6, 7, 10, 11

7. A 200 N-mm couple acting counterclockwise keeps the member in equilibrium while it is subjected to other forces acting in the plane (shown schematically at the left). The four dots denote equally spaced points along the members.

Assuming the other forces stay the same, what load(s) could replace the 200 N-mm couple and maintain equilibrium?

Analysis of Student Responses: Two of the five students responded with the correct answer E. These two students were able to identify that both the force and moment of the selected response are equivalent to the given situation.

**Student A:** “...it’s a couple moment because it has equal forces in opposite directions and so you’re just going to take the distance between the two which is going to be a moment, so you take your 50 mm times your 4 Newtons and that’s going to give you your 200 Newton mms”

**Student C:** “I’m liking choice E,... because the X, the Y forces cancel. The forces cancel each other in terms of up and down, left and right, just like the original ... Moments about B has to be equal to 200 Newton meters-- Newton millimeters. And the sum of the forces on the object in the Y direction have to be zero. And the sum of the force in the X direction all have to be zero. All of these are conditions that have to be satisfied in order for the 200 Newton millimeter force to be preserved.”

Two of the remaining students were able to articulate a need for either balanced forces or moments, but were unable to determine that answer E provided both balanced forces and an equivalent moment.

**Student B:** “... for option B, I did 100mm times the force, 2 Newton’s, and I get 200 Newton mms about the second point and I think that’s that would be the equivalent moment.”

**Student D:** “... I remember that couples are equal and opposite forces, so that’s-- that was the main feature that I was thinking about, I guess, which led me to pick answer D.”

Student B was able to select a response with an equivalent moment, but did not recognize that this response would lead to unbalanced forces. This error corresponds with Common Error 10: failure to impose balance of forces in all directions and moments about all axes. Although Student D seemed to recognize the need for balanced forces, the selected response demonstrates an improper accounting of a couple in the moment summation. This error aligns with Common Error 11.
The fifth student initially selected the correct response E and provided some evidence of a considered equivalent force, but subsequently flip-flopped between responses A and E.

**Student E:** “I think it’s E... I think it’s a couple. So a couple is the formula at R cross F, it’s given to be 200. So-- and here, the distance is 50. It’s acting on the second point. So the distance from the first point is 50. So I found out the value for F to be four Newtons, acting at this distance to produce that couple. And there was no option with four Newtons at this point, right... Let’s see. This is 400 acting at a distance of 400 divided by 100 is four Newtons. I think it’s-- it could be A, actually.”

In selecting response A, the student fails to apply the concept that moments are equivalent at any point on the body. This provides evidence that the student does not understand Cognitive Attribute 21, which includes the concept that the moment exerted is the same about any point. Additionally, this error has been included in Table 5 as a possible Common Error 13.

**Table 5. Summary of findings for Item 2 (Q-7)**

<table>
<thead>
<tr>
<th>Experts' Common Errors</th>
<th>Errors in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failing to account for the mutual (equal and opposite) nature of forces between connected bodies that are separated for analysis.</td>
<td>6</td>
</tr>
<tr>
<td>Ignoring a couple that could act between two bodies or falsely presuming its presence.</td>
<td>7</td>
</tr>
<tr>
<td>Failure to impose balance of forces in all directions and moments about all axes.</td>
<td>10</td>
</tr>
<tr>
<td>Having a couple contribute to a force summation or improperly accounting for a couple in a moment summation.</td>
<td>11</td>
</tr>
<tr>
<td>Student fails to recognize that location of moment is irrelevant; the moment about any point on a rigid body is equivalent</td>
<td>13*</td>
</tr>
</tbody>
</table>

*Note: Errors generated from student responses identified with *. 

**Item 3: Q-17**

**Cognitive Attribute: Contact Forces (C.A. 5)**

17. The circular portion of part 1 contacts the curved position of part 2. Neglect friction between part 1 and part 2. Consider the bodies to be in the orientations shown.

Could the reactions of part 2 on part 1 be as shown?
Analysis of Student Responses: Three of the five students were able to correctly respond by selecting answer D, and identified that the contact force would occur at the point of contact and perpendicular to the surface, thus demonstrating mastery of Cognitive Attribute 5.

**Student C:** “...forces exerted by contacting things are called normal forces because they are normal. So they have to be along that axis.”

**Student D:** “...it’s always going to cause a force perpendicular to the contact point between the two parts...”

**Student E:** “Because I think the reaction force is the normal force and normal force needs to be normal to the point, like the point of contact.”

The remaining two students selected the incorrect response C. Both students indicated the force occurring at the point of contact; however only one of these students explicitly indicated that this would include a non-normal contact force.

**Student A:** “...what I believe would be a vertical contact force at that contact point. Because it’s the direction that the force is pushing up on part 2.”

**Student B:** “Okay so I choose C... there’s a force acting on part 2, so part 2 is trying to push it counter clockwise and there would be a force at the contact point.”

For this item both student responses align with Common Error 8, as the students do not sufficiently restrict the possible forces acting at the contact point to those perpendicular to the contact plane. However as presented in Table 6, there is no identified evidence in the student transcripts of the additional expected Common Errors 2 and 3.

Table 6. Summary of findings for Item 3 (Q-17)

<table>
<thead>
<tr>
<th>Experts’ Common Errors</th>
<th>Errors in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two.</td>
<td>2 No identified evidence</td>
</tr>
<tr>
<td>3 Leaving a force off the free body diagram (FBD) when it should be acting.</td>
<td>3 No identified evidence</td>
</tr>
<tr>
<td>8 Not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces.</td>
<td>8 Student selected a response that allows for a non-normal contact force.</td>
</tr>
</tbody>
</table>

Item 4: Q-23

Cognitive Attribute: Friction Forces (C.A. 7)

22. Two blocks are stacked on top of each other on the floor. The friction coefficient is 0.2 between all contacting surfaces (Take this to be both static and kinetic coefficient of friction). Then, the horizontal 10 N force is applied to the lower block.

What is the horizontal component of the force exerted by the floor on the lower block?

(a) 4 N  
(b) 6 N  
(c) 8 N  
(d) 10 N  
(e) 18 N
**Analysis of Student Responses:** Only one of the five students was able to identify the friction force for the given situation and correctly respond with answer D.

**Student B:** “Because first I determined if the block is going to move or not; if the block is moving then the friction between the block and the floor will equal to the maximum friction, which is 18 Newton’s, but in this case the force is, the 10 Newton’s is not large enough to let the blocks to start moving, so I think it’s 10 Newton’s.”

Two of the five students fell into the trap of responding with the maximum static friction force. These students demonstrate Common Error 9 by presuming that the friction force is at the slipping limit (µN), even though equilibrium is maintained with a friction force of lesser magnitude.

**Student A:** “Your pushing force is going to be your 10 Newtons going to the left, and then your frictional force to counteract that is going to be to the right but you need to use the F=µN equation to find that...”

**Student C:** “So friction, force is proportional to the normal force by the coefficient of friction. So we’ve got friction force of the bottom equals 0.2 times the normal force. I have to find to normal force, right. So you can do a force balance and know that here that we have a total of 90 Newtons coming down and no other forces. So 90 Newtons have to come back up. So that’s the 90 Newtons that we use and we have 0.2 times 90 equals 18.”

The remaining two students experienced difficulty with the two components of the system and seemed to be unable to recognize how treating the two blocks as one system would simplify their calculations. In this manner, they demonstrated Common Error 2: failure to take advantage of the options of treating a collection of parts as a single body.

**Student D:** “... I guess if we do treat the whole thing as a system, then it would be D... Just by doing the sum of the forces in the X have to equal zero because the friction, or as they call it the horizontal component of the force exerted on the floor, that would have to be exerted on the floor by the block. I hadn’t thought about it like... Yeah, so then it would be D just by doing the sum of the forces in the X direction. But I guess answers A and C could also be true if that 60 Newton block stays stationary because then, like, let’s say there’s a two Newton frictional force between the two blocks, then do the sum of the forces in the X, and you’re going to have a 10 going to the left, and then you can have a two and an eight going to the right, or you can have a four and a six going to the right or a six and a four going to the right, so that kind of leaves A, B, and C.”

**Student E:** “Because I first drew two free-body diagrams. I named the 16 year from one block A and, this one, block B, the lower one is B, so I drew their free-body diagrams with their weights acting downwards and the normal force acting upwards and then the frictional forces depending on the direction of motion on the lower block, so the lower block is moving in the left direction, the frictional force acting on the lower block would be in the right direction, so this is FB, and, because this block moves here, the upper block is going to move to the right, it’s going to tend to move to the right, so the frictional force is going to be like that for this individually, so I did the summation of forces in the X and Y directions, got what the normal forces acting on block A and B are, and then I summed the forces in the X direction for the lower block and found the value.”

Although Common Error 9 was the only expected error, the two students provide evidence supporting the inclusion of Common Error 2 as presented in Table 7.
Table 7. Summary of findings for Item 4 (Q-23)

<table>
<thead>
<tr>
<th>Experts' Common Errors</th>
<th>Errors in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presuming a friction force is at the slipping limit ($\mu N$), even though equilibrium is maintained with a friction force of lesser magnitude.</td>
<td>2 Student does not take advantage of treating the collection of parts as a single body</td>
</tr>
<tr>
<td>Student fails to recognize that the resulting friction force is less than or equal to the maximum, and selects the friction force at slipping limit</td>
<td>9</td>
</tr>
</tbody>
</table>

Item 5: Q-27
Cognitive Attribute: Couples & Equilibrium (C.A. 21)

27. The forces act at points indicated. All magnitudes are greater than zero, and the forces act in the directions and senses shown.

Which one of the following additional loads, if given the right magnitude, could lead to equilibrium?

Analysis of Student Responses: Four of the five students correctly responded with answer A; either students indicated that the sum of the forces had to be equal or included a slight indication of this. Students did not indicate that the moment is the same about any point as described in Cognitive Attribute 21, however most students considered moments when responding.

Student A: “Moment wouldn’t really matter in this case for equilibrium because that’s just a couple force that’s going in equal and opposite directions. I think it’d be A to counteract with the angled force that’s coming at P already so that the X and Y components of the first force at P can counteract with the new force at P to create equilibrium between an object.”

Student B: “So I think if I add another force at P, so the entire block will have no moment, but only have forces.”

Student C: “I selected A because it’s-- I tested them. I just looked at them all and I saw that all of them- all the other ones either left it not balanced, the forces. Left the forces not balanced on the member or left the moment about one of the points.”

Student E: “Yeah, resolving the force acting at P into its X and Y components and seeing how equilibrium could be attained in a way that the forces in the opposite direction would cancel.”

The remaining student initially chose response C, then changed to D.

Student D: “so to compensate for the force that’s pointing down to the right, you already have one that’s pointing to the left. So now you need one that goes up, ... now I’m second-guessing myself again because there’s distance between them”

Student D: “...you could break up P into just an X component, and because only that X component acts as a moment at R and Q because the other-- the Y component of P would point right through R, so that
wouldn’t create any moment, so you have an X component at P, that would create a moment there. Then you have a Q or a-- and an X component and a Y component at Q-- no, no, no. There is no Y component, so there’s only an X component, so, yeah, I guess a moment at R would be sufficient, I believe.”

Although the student initially seemed to recognize the need for balanced forces, by selecting an incorrect response, the student demonstrated Common Error 10: failure to impose balance of forces in all directions and moments about all axes. This evidence of Common Error 10 was the only error identified as presented in Table 8. The expected errors included Common Errors 7, 10, and 11. Even though this item was expected to be the most difficult, only one student incorrectly responded to the problem and there was limited evidence of Common Errors.

Table 8. Summary of findings for Item 5 (Q-27)

<table>
<thead>
<tr>
<th>Experts’ Common Errors</th>
<th>Errors in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Ignoring a couple that could act between two bodies or falsely presuming its presence.</td>
</tr>
<tr>
<td>10</td>
<td>Failure to impose balance of forces in all directions and moments about all axes.</td>
</tr>
<tr>
<td>11</td>
<td>Having a couple contribute to a force summation or improperly accounting for a couple in a moment summation.</td>
</tr>
</tbody>
</table>

Discussion

How do students’ conceptions and misconceptions align with expected skills and mastery profiles of the Concept Assessment Tool for Statics (CATS)?

An analysis of the student interviews provided some evidence that students’ conceptions and misconceptions aligned with expected cognitive attributes, as was previously established by the Fusion Model, because students committed some of the common errors expected for each item. However, none of the sampled students’ response behavior across the selected items aligned with the top five mastery profiles as seen in Table 2. This is not surprising as only one of the cognitive attributes (CA 10, pin on slot) listed in the profiles was included in the pilot study. Instead, the pilot protocol focused on cognitive attributes that were found to be more difficult by experts and had the highest occurrence of non-mastery as determined by the Fusion Model (Table 2). From the pilot study results, the highest occurrence of non-mastery was seen with CA 7, friction force, with the next highest occurrence of non-mastery seen in the item addressing CA 1, equivalence and CA 21, couples & equilibrium. The sampled students performed equally on items addressing CA 10, pin on slot and CA 5, contact forces. Although expected to be the most difficult item in the interview, the highest percentage of correct responses was found with item 5, which addressed CA 21, couples & equilibrium. Given the small sample size of the students interviewed, further study is needed to determine if student response behavior aligns with CATS mastery profiles.

Of the eight expected Common Errors, six were identified in student interview transcripts (2, 3, 8, 9, 10, and 11), while no evidence was found of the remaining two Common Errors (6, 7). As seen in Table 9, errors identified in the student interviews correspond with some of the expected Common Errors as determined by domain experts. Additionally, two errors emerged from student interviews that may be considered for inclusion as expected Common Errors for future studies. In two instances (Items 1 & 2), students were able to select the correct answer among
distractors while simultaneously providing indication of one of the expected common errors. Further study may be necessary to determine what effect, if any, results from this behavior.

Table 9. Summary of Common Statics Errors

<table>
<thead>
<tr>
<th>CATS Item</th>
<th>Experts' Common Errors</th>
<th>Errors Identified in Student Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Q-13)</td>
<td>2, 3, 8</td>
<td>3, 8, other</td>
</tr>
<tr>
<td>2 (Q-7)</td>
<td>6, 7, 10, 11</td>
<td>10, 11, other</td>
</tr>
<tr>
<td>3 (Q-17)</td>
<td>2, 3, 8</td>
<td>8</td>
</tr>
<tr>
<td>4 (Q-23)</td>
<td>9</td>
<td>2, 9</td>
</tr>
<tr>
<td>5 (Q-27)</td>
<td>7, 10, 11</td>
<td>10</td>
</tr>
</tbody>
</table>

These findings support the strength of CATS as a diagnostic tool, which may be used to better understand students’ conceptual thinking. In accordance with Minstrell’s argument\(^7\), instructors armed with tools, such as concept inventories, that provide insight into students’ thinking, may be better equipped to target instruction for increased cognition.

**Conclusions and Implications**

Concept inventories like CATS may be used to inform instructors of students’ prior knowledge and diagnose common misconceptions held by students. However, the diagnostic capacity should be validated with qualitative evidence supporting the expected conceptions and misconceptions. In this study, a small sample of students were interviewed to provide evidence of thinking that may be used to support the expected cognitive attributes addressed by CATS as well as expected common errors regarding those concepts as identified by domain experts. An obvious limitation of the study involves the small sample of student interviews. As this study was intended to pilot interview protocol for use in further studies, selecting only a small number of participants proved manageable within the constraints of the pilot study. Additional work that provides further evidence of student thinking is necessary to validate the diagnostic capabilities of CATS. That work is now underway.

Beyond the study limitations due to number of participants, there are concerns that stem directly from the design of CATS items. Recommendations for concept inventory design suggest that items should isolate and assess single concepts\(^2\), however in the case of Item 2 (Q-7) mastery of two cognitive attributes: CA 1, equivalence, and CA 21, couples & equilibrium, is necessary for a correct response. For this item, evidence suggests that students make a variety of errors and further study may be needed to understand the effects of items that include multiple concepts. Even when items address a single concept, pilot study evidence suggests that students may be able to attain a "correct" answer with only incomplete or intuitive understanding of that concept. However, the high diagnostic capacity of CATS as determined through application of the Fusion Model may overcome these limitations.\(^1\)

As students' underlying thinking is not directly evident from selected responses, i.e. multiple-choice assessments, interview protocols are necessary that explicitly encourage students to explain why selected responses are correct and remaining answers are incorrect. The protocol used for this study encouraged students to explicitly state why the answers they selected were correct and other answers incorrect, however the explanations that students provided did not
always match exactly with expected conceptions or misconceptions identified by experts. In these cases, the analysis of student explanations may be influenced by interpretations of what students indicate or intend. It is difficult to get students to verbalize their conceptual thinking without relying on expert knowledge to frame student misconceptions. With this difficulty in mind, some modifications to the protocol were developed. These modifications included emphasizing that students were providing evidence to evaluate the test items, and not the students’ performance on the problems as well as encouraging students to verbally explain any physical actions they may perform, including any hand motions, drawings or answer selections. Recommendations for the interviewer included saying as much as possible aloud to ensure capture on the transcript, avoiding specialized terminology when prompting students for explanations and listening for evidence of cognitive attributes and common misconceptions when guiding interviews. Further iterations of protocol design may aid in understanding how to best elicit students’ conceptual thinking in a manner that does not rely on expert knowledge to frame students’ conceptions.

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Bibliography

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APPENDIX - CATS Protocol

“During this session/interview/time, I will ask you to solve/answer some problems. After you solve the problem, I will ask you to explain your answers. I may also ask you follow-up questions about your explanations. I am not looking for a particular answer and these problems are not meant to trip you up. I simply want to better understand your answers.”

“Do you have any questions before we begin?”

(Do I need to say anything about the session being recorded and I will occasionally take notes?)

1. Begin the tape
2. State interview #, name of interviewee and interviewer(s), date
3. Give the student the problems in the following order: 13, 7, 17, 23, and 27.

Question 13
Please tell me why you selected your response. Please explain your answer.

Probes:
1. I am not sure that I fully understand. Please give me some more detail/please tell me more.
2. Please tell me why you did not select the other responses. (The student can go through and explain why the other responses were not chosen) Probe: Why do you think/believe that is not correct? What led you to believe/think that?
3. Were there any other responses that also appeared correct? Were there any other options that you were considering?
4. Please walk me through what happens to the mechanism once the downward force acts. (May point to question diagram)
Question 7
Please tell me why you selected your response. Please explain your answer.

Probes:
1. I am not sure that I fully understand. Please give me some more detail/please tell me more.
2. Please tell me why you did not select the other responses. (The student can go through and explain why the other responses were not chosen)
   Probe: Why do you think/believe that is not correct? What led you to believe/think that?
3. Were there any other responses that also appeared correct? Were there any other options that you were considering?
4. Please walk me through what happens to the mechanism once the upward force acts. (May point to question diagram)

Question 17
Please tell me why you selected your response. Please explain your answer.

Probes:
1. I am not sure that I fully understand. Please give me some more detail/please tell me more.
2. Please tell me why you did not select the other responses. (The student can go through and explain why the other responses were not chosen)
   Probe: Why do you think/believe that is not correct? What led you to believe/think that?
3. Were there any other responses that also appeared correct? Were there any other options that you were considering?
4. Please walk me through what happens to the mechanism once the upward force acts. (May point to question diagram)
Question 23
Please tell me how you determined your answer/ Please tell me how you determined the number (depends on answer choice).

Probes:
1. I am not sure that I fully understand. Please give me some more detail/please tell me more.
2. Please tell me why you did not select the other responses. (The student can go through and explain why the other responses were not chosen) Probe: Why do you think/believe that is not correct? What led you to believe/think that?
3. Were there any other responses that also appeared correct? Were there any other options that you were considering?
4. Why is your calculation correct for this situation?

Question 27
Please tell me why you selected your response./Please explain your answer.

Probes:
1. I am not sure that I fully understand. Please give me some more detail/please tell me more.
2. Please tell me why you did not select the other responses. (The student can go through and explain why the other responses were not chosen) Probe: Why do you think/believe that is not correct? What led you to believe/think that?
3. Were there any other responses that also appeared correct? Were there any other options that you were considering?
4. Please walk me through what is happening to the object as the forces act at the points indicated. (May point to question diagram)
Part 2- These questions will only be asked after all five questions have been covered with the participant if time permits.

Question 13
1. Why should the force act at 20 degrees above the horizontal and not 20 degrees counterclockwise to the vertical? (May point at B and D)
2. Why does/does not pin A also have a moment? (May point at E)

Question 7
3. Why does an upward load of 2N at the last point maintain equilibrium? (May point at B)
4. Why do a downward load of 200N at the first point and an upward load of 200N at the third point maintain equilibrium? (May point at D)
5. Why do a downward load of 4N at the third point and an upward load of 200N at the last point maintain equilibrium? (May point at E)

Question 17
6. Why is I possible/impossible? (May point to figure I)
7. Why is II possible/impossible? (May point to figure II)
8. What other information is needed to answer this question? (if E chosen)

Question 27
9. Why would an upward directed load at point P lead to equilibrium? (May point at A)
10. Why would a moment at point R lead to equilibrium? (May point at C)
11. Why would an upward directed load at point Q lead to equilibrium? (May point at D)