AC 2007-2551: VARIATION OF FRACTAL DIMENSION OF LEAVES BASED ON STEM POSITION

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Variation of Fractal Dimension of Leaves Based on Stem Position

Abstract

Utilization of methods based on Euclidean geometry to perform routine measurements of irregular objects could prove to be exceptionally difficult and particularly inefficient. These irregular arrangements such as leaf shapes are called fractals and are more efficiently described within the geometry of fractals.

The purpose of the experiment, in the present study, is to examine shapes of plant leaves in relation to their position on the stem in terms of fractal dimensions. The hypothesis suggests that fractal dimension does vary among the leaves located at various positions on the stem.

In this experimental study, five samples of Norfolk Island Pine Araucaria Heterophylla plants were obtained and were carefully deprived of their leaves. The fractal dimension of each leaf was determined using the box-count method. Five trials were conducted using five plants. The mean fractal dimensions of each leaf was obtained and then analyzed by ANalysis Of VAriance between groups [ANOVA].

I. Introduction

Shapes have always been an important aspect in biological systems. Although usually ignored, shapes play a major role in description of functions of various organisms. Traditionally, the shapes of objects and organisms have been described using Euclidean geometry[1]. Euclidean geometry describes the basic, regular figures that are most familiar such as lines, squares, cubes, etc. Irrespective of the case, all these structures have dimensions that are positive integers (whole numbers): 0 for a point, 1 for a line, 2 for a surface and 3 for volume[2]. However, objects do not always display these simple shapes, especially in nature.

The study of fractal dimension is currently being applied to almost every branch of science, mathematics[3] and economics[4]. Its applications to medical and biological sciences have been extensive. Recent studies have shown that fractal geometry can be useful for describing the pathological architectures of tumors and, perhaps more surprisingly, for yielding insights into the mechanisms of tumor growth, i.e., angiogenesis, that complement those obtained by modern molecular methods[5]. In other cases, the study of fractal dimensions in dynamic systems such as the fluctuations of a human heart beat could lead to the detection of heart diseases depending on the irregularity of the heart beat frequency[6]. These fractal dimensions could be measured using the box count method.

Fractals are used in many applications across the sciences. One of the biggest advances in fractal application has been in the fields of image analysis and pattern recognition. From understanding facial expressions to modeling a pattern, fractals have made significant advancements.

In a paper by Iftekharuddin, et. al.[7], fractals have been applied to brain tumors. Using three methods identified as piecewise modified box-counting, piecewise triangular prism surface area, and piecewise threshold box counting, images of the brain have been analyzed. The difference in the images between the normal brain and tumor brain are seen in both fractal dimension and intensity histogram. This allows for both detection and location of tumors in a brain.

Another group of researchers have used fractals to distinguish different dietary habits in pre-historic animals. At the University of Arkansas, Peter Ungar[8] has studied the micro wear of animal teeth and its association with the plants consumed. Using fractal analysis software, the indentations on modern animal teeth were found to have specific patterns based on its diet. During attempts of tracing these back to pre-historic fossils, a change in diet from fruits and nuts to grass was identified thus providing another link to a long chain of evolution.

Fractal analysis tools are used widely to identify trends in natural geography and shape. Abstract structures such as the structure of the Internet have been modeled[9]. Fractals have also been useful to model shapes of coastlines and galaxy[10].

II. Box Count Method and Fractals

In the Box Count method, a fractal image is superimposed by a grid and the boxes covered by the image are counted. This step is repeated by using a decreased size of grid (boxes). The data is then converted to a scatter graph that has the axes for the size of the box and the number of the boxes counted. If the points form a straight line, the image would be considered to be a fractal. The slope of the line represents the fractal dimension of that image.

Further understanding of fractal dimensions, in this case, shapes, will benefit various branches of science since understanding shapes can predict vital functions of components in biological systems. But, in order for this to be accomplished, there is a need to consider, initially, a system that is rather simple and plain, such as the understanding of the usage of fractal dimension to examine the event of shape change in leaves of plants in relation to their position on the stem of the plant.

Fractals, in the most general definition, are simply self-similar structures. In this sense, fractals are all around us in the shapes of a coastline, a fern, a tree, or a mountain range. A tree, for instance, is a trunk with branches and leaves, while a branch has twigs and leaves. Hence, the smaller parts of a tree appear to have the same structure as the whole. Until Benoit Mandelbrot[3], Gaston Julia[11] and Pierre Fatou[12] discovered self-similar structures in iterative mappings, such structures had gone largely unnoticed. Beginning in the late 1910's and into the 1920's, Julia[11] and Fatou[12] led the study of these self-similar structures. At that time, there were no computers to produce the images that we see today. Consequently, interest in fractals was restricted to those very few individuals who could in some sense understand the mathematics behind the pictures that are drawn today.

Although Mandelbrot[3] invented the word *fractal*, many of the objects featured in *The Fractal Geometry of Nature* had been previously described by other mathematicians (the Mandelbrot set[3] being a notable exception). However, they had been regarded as isolated curiosities with unnatural and non-intuitive properties. Mandelbrot[3] brought these objects together for the first time and highlighted their common properties, such as self-similarity (sometimes partial or statistical), scale invariance and (usually) non-integer Hausdorff dimension.

Mandelbrot[3] also emphasised the use of fractals as realistic and useful models of many natural phenomena, including the shape of coastlines and river basins; the structure of plants, blood vessels and lungs; the clustering of galaxies; Brownian motion; and stock market prices. Far from being unnatural, Mandelbrot[3] held the view that fractals were, in many ways, more intuitive and natural than the artificially smooth objects of traditional Euclidean geometry.

In mathematics, the **Hausdorff dimension**[3] is a positive real number associated with any metric space. It was introduced in 1918 by the mathematician Felix Hausdorff[3]. Many of the technical developments used to compute Hausdorff dimension for highly irregular sets were obtained by Abram Samoilovitch Besicovitch. For this reason, Hausdorff dimension is sometimes referred to as **Hausdorff-Besicovitch dimension**. It is also less frequently called the **capacity dimension** or **fractal dimension**.

It should be noted that there are various closely related notions of possible fractional dimensions. For example, the box-counting procedure generalizes the idea of counting the number of squares of graph paper in which a point *X* can be found, as the size of the squares is made smaller and smaller. In many cases, these notions overlap but the relation between them is highly technical, although empirical.

Box-counting dimension is a simple way of estimating the Hausdorff dimension[3] for fractals. It involves computing the box-counting dimension from a grid that is superimposed on a fractal image and counting the number of boxes in the grid that contains part of the fractal. Then the number of boxes in the grid is increased (but covering the same area: the boxes get smaller) and the boxes are counted again. If the number of boxes in the first and second grids is G_1 and G_2, and the counts are C_1 and C_2, then a dimensional parameter D is defined by the formula:

$$D = \frac{\log\left(\frac{C2}{C1}\right)}{\log\left(\sqrt{\frac{G2}{G1}}\right)}$$

Fractal geometry facilitates the measurement of irregular shapes by comparing its dimensions from one scale to another. In Euclidean geometry, units of measurements such as inches and meters are used; in fractal geometry fractal dimensions are used.

III. Purpose

Specimens in biological systems such as plants exhibit various structures, or more simply, shapes. First of all, shapes are used to distinguish or describe the differences between species. Secondly, shapes play a major role in the physical functions of organisms. Fractal dimensions are numerical measurements of nonstandard shapes. Fractal dimensions could be described as dimensional values $1<D<2$ (since standard shapes have dimensions that are whole numbers; 1 for a line, 2 for a square, etc.). They can be employed to determine the structure of any object regardless of the irregularity of shape.

The purpose of this study is to examine the shape of plant leaves in relation to their position on the stem. The change in shape of leaves, if any, will be demonstrated by mathematical evidence. Fractal dimensions of the leaves shall be calculated in order to determine the degree of variation in their shapes.

IV. Experimental Data, Results and Discussion

Five samples of Norfolk Island Pine Araucaria Heterophylla were obtained. The top five leaves were labeled from 1 to 5 with 1 as the top leaf. Each leaf was placed on different sized graphing papers; 1 inch, ½ inch, ¼ inch and ⅛ inch which are, in metric units; 2.54 cm, 1.27 cm, 0.635 cm and 0.3175 cm, respectively. The number of boxes is counted for each size of graphing paper and then recorded. Then a log - log plot is constructed using the recorded data. The X-axis is made up of log(1/size of box); the Y-axis is made up of log(1/the number of counted boxes). A set of data points is formed and the best-fit line is generated. The slope of that line is the fractal dimension of that leaf. The process is repeated for the rest of the leaves. The procedure is repeated for the rest of the plants.

In Figure 1(a), an image of one of the Norfolk Island pine plants, which was considered in this study, is presented. A sample leaf is shown in Figure 1(b).

Figure 1 (a) Image of the Norfolk Island pine plant; 1(b) A sample leaf

The fractal dimensions were determined for the irregular shaped Norfolk Island pine leaves (Araucaria Hetrophylla). As can be seen in Figures 2(a) – 2(e), the fractal dimension varies by its position on the stem of the plant. In determining fractal dimensions, it was found that leaves that have positions in the middle along the stem exhibit higher fractal dimension than the topmost or the bottommost leaf. The statistical analysis, ANOVA (analysis of variance) [13] was also performed in order to test the null hypothesis: the mean fractal dimension among leaves 1, 2, 3, 4 and 5 are not different. The experimental results of the comparison of mean fractal dimension (FD) for each leaf position is summarized in Figure. 3.

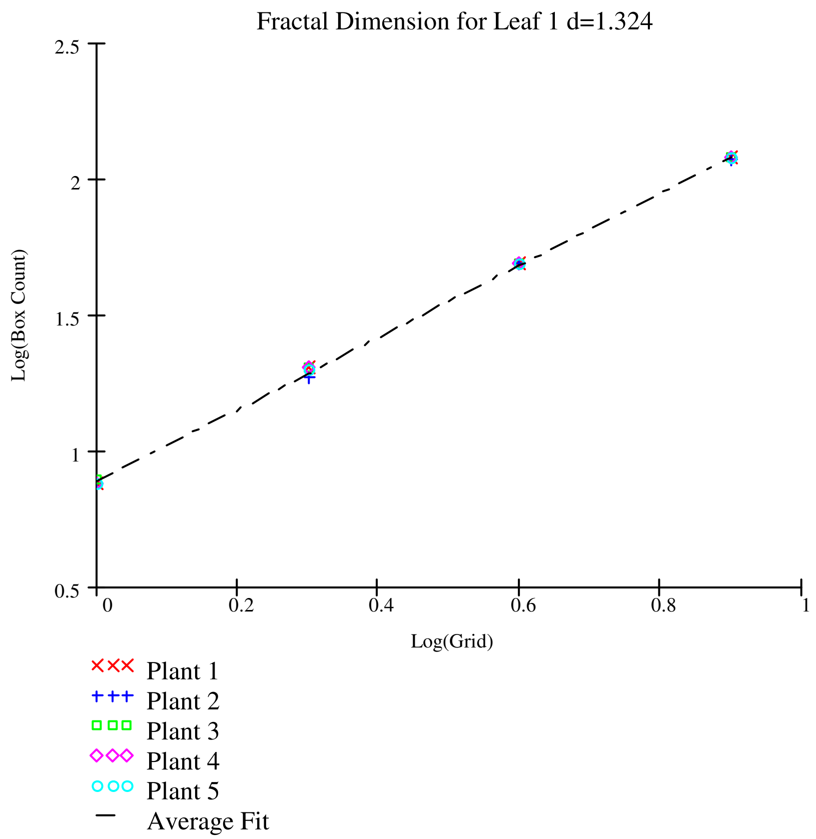


Figure 2 (a)

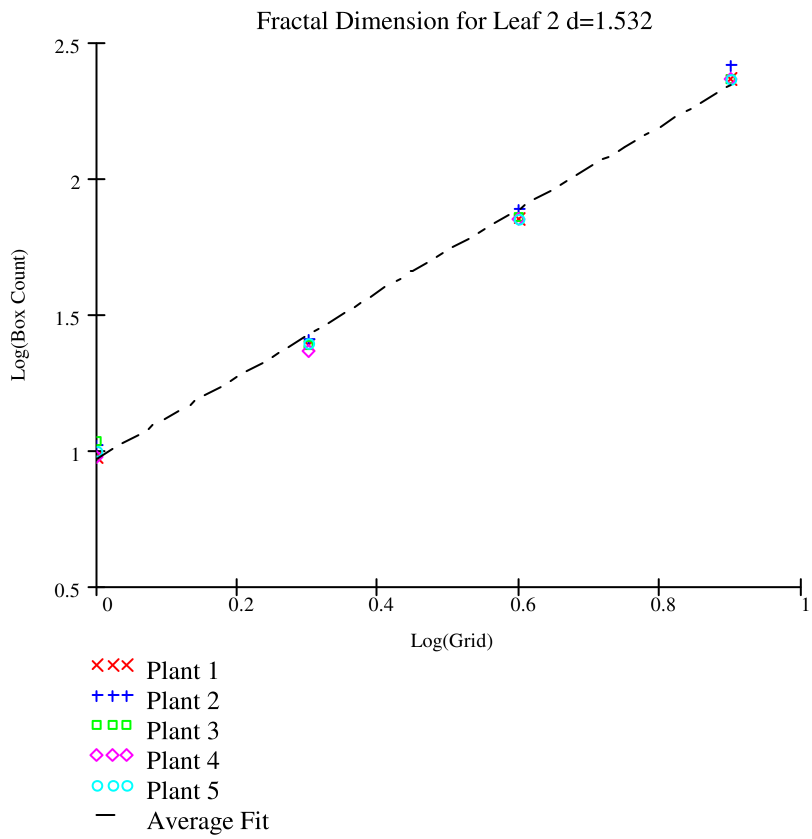

Fractal Dimension for Leaf 2 d=1.532
Log(Box Count)
Log(Grid)
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
Plant 1
Plant 2
Plant 3
Plant 4
Plant 5
Average Fit

Figure 2 (b)

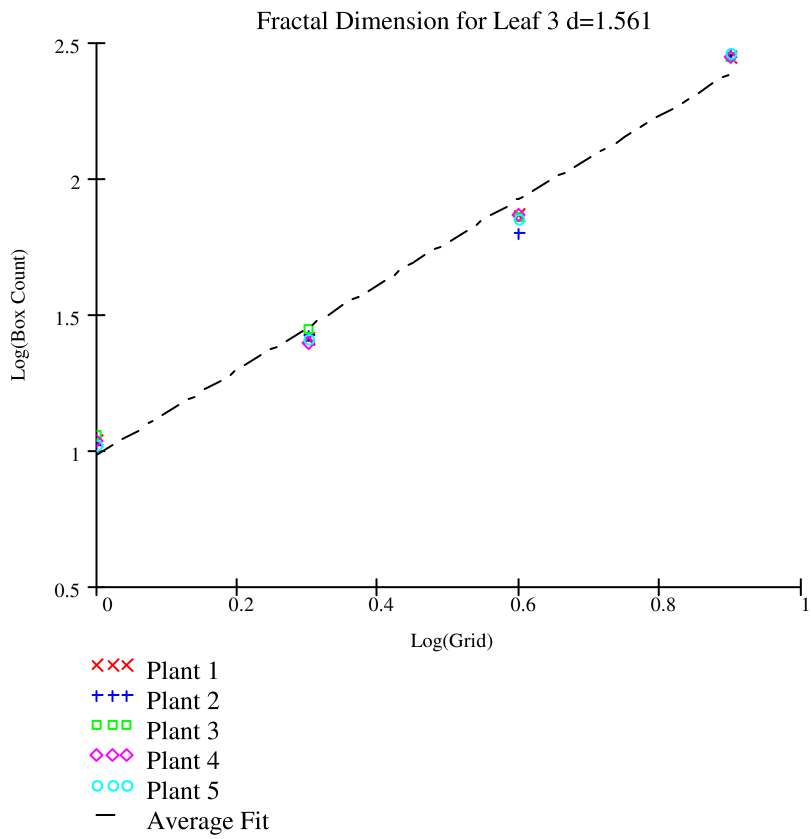
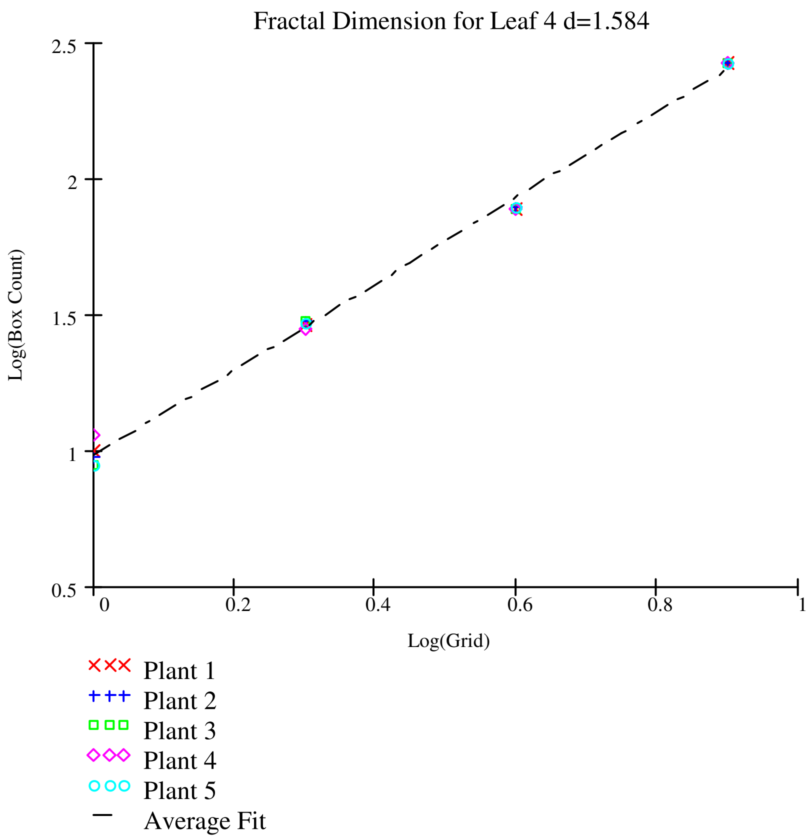

Fractal Dimension for Leaf 3 d=1.561
Log(Box Count)
Log(Grid)
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
Plant 1
Plant 2
Plant 3
Plant 4
Plant 5
Average Fit

Figure 2 (c)



Fractal Dimension for Leaf 4 d=1.584
2.5
2
1.5
1
0.5
Log(Box Count)
0
0.2
0.4
0.6
0.8
1
Log(Grid)
Plant 1
Plant 2
Plant 3
Plant 4
Plant 5
Average Fit

Figure 2 (d)

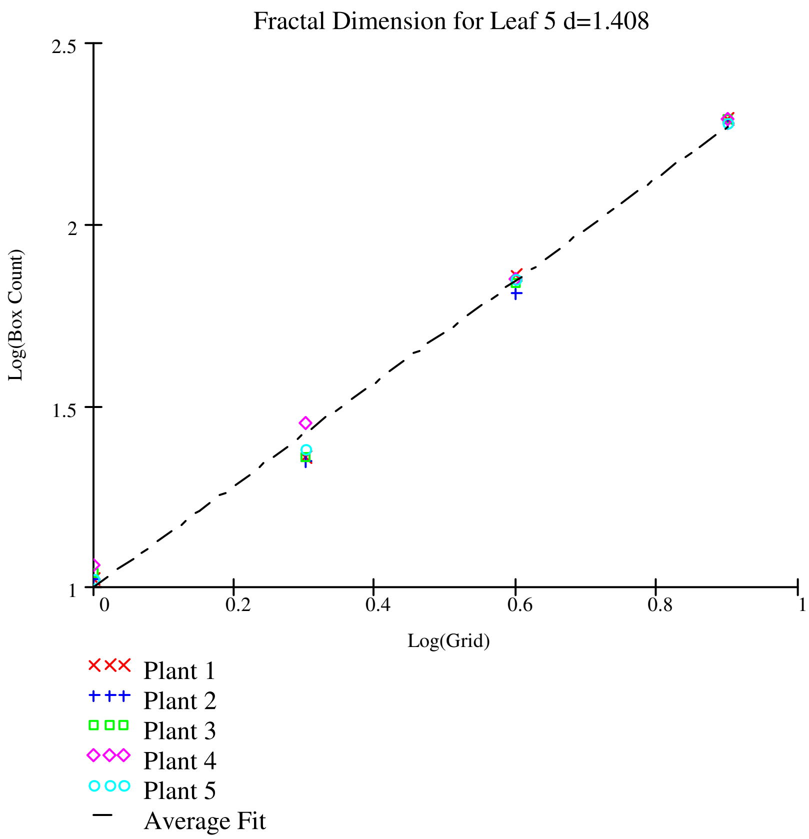

Fractal Dimension for Leaf 5 d=1.408
Log(Box Count)
Log(Grid)
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1
Plant 1
Plant 2
Plant 3
Plant 4
Plant 5
Average Fit

Figure 2 (e)

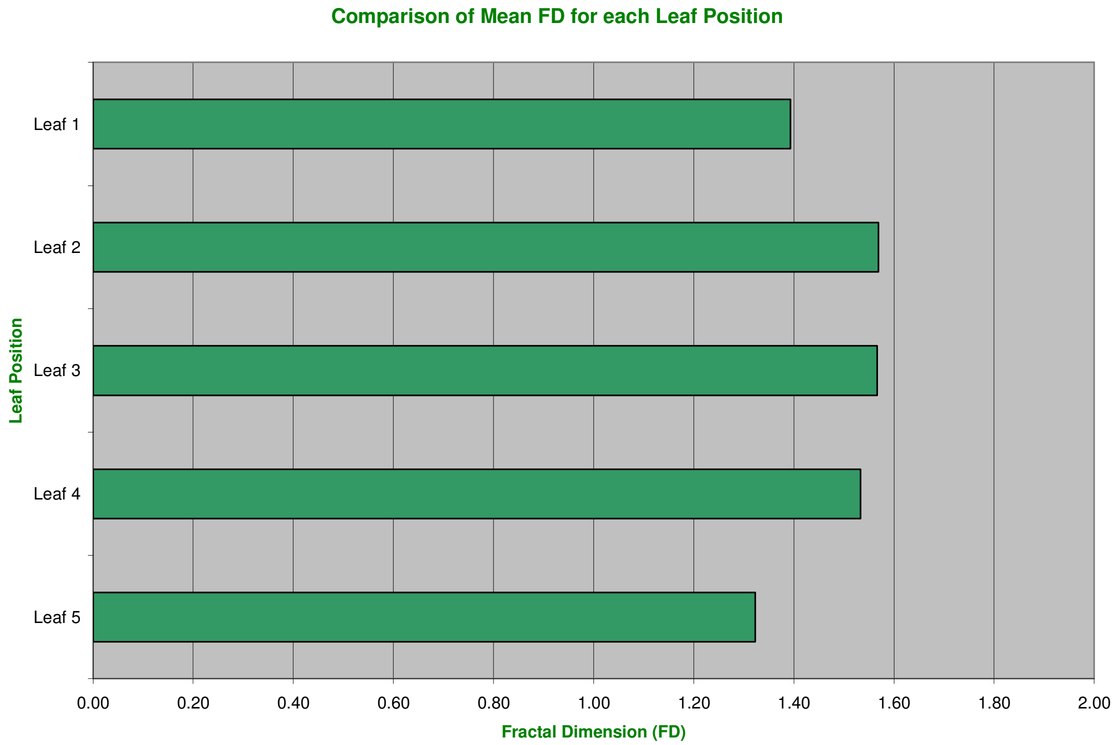


Figure 3

The results of the statistical analysis have shown the shape of leaves in terms of fractal dimension. For number of degrees of freedom, df = 4 between leaves, 20 within leaves, the f ratio, f = 2.87, the calculated value of df = 84.4 > 2.042 is significant at the 0.05 level. As is well known in statistical analysis, the significance level for 0.05 is less than the significance level for 0.001. The null hypotheses that the mean fractal dimension of leaves 1 through 5 are equal to each other was therefore rejected. Because the null hypothesis was rejected at the 0.05 level of significance, the research hypothesis that fractal dimension does change in relation to leaf position was supported.

Note that when one utilizes ANOVA as a form of statistical test, there are two degrees of freedom: df within groups, and df between groups. This is because of the fact that, in ANOVA, unlike the T-test in which one has to compare two groups at one time, all the experimental groups are simultaneously compared.

Conclusions

An experimental study has been performed to analyze the shape of leaves as a function of their position on the stem in Norfolk Island pine (Araucaria Hetrophylla). It has been found that fractal dimension does differ along the stem of the plant. It was also observed that the leaves in the middle have higher fractal dimensions. A statistical test based on ANOVA showed that at df = 4 between leaves, 20 within leaves, f = 2.87, the calculated value of df = 84.4 > 2.042 is significant at the 0.05 level, and rejects the null hypothesis that the fractal dimension of leaves are not significantly different. As a result, the experimental data supports the hypothesis that fractal dimension does differ among leaves due to varying stem positions.

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References:

1. G. Hartvigsen, The analysis of leaf shape using fractal geometry, American Biology Teacher, 62(9): 664-669, (2000).

2. M. Alfonseca, A. Ortega, Determination of fractal dimensions from equivalent L systems, IBM Journal Research & Development, Vol. 45, pp 797-805, Nov (2001).

3. Benoit B. Mandelbrot, The Fractal Geometry of Nature, W.H. Freeman and Company, New York (2000).

4. Edgar E. Peters, Fractal Market Analysis, John Wiley & Sons, New York (1994).

5. J.W. Baish and R. K. Jain, Fractals and Cancer, Cancer Research, Vol. 60, pp 3683-3688, July (2000).

6. A.L Goldberger, L.A.N. Amaral, J.M. Hausdorff, P.C. Ivanov and C.K. Peng, Fractal Dynamics in Physiology: Alterations with disease and aging, PNAS, Vol. 99, pp. 2466-2472, Feb (2002).

7. K.M Iftekharuddin, et. al., A fractal analysis approach to identification of tumor in brain MRimages, Engineering in Medicine and Biology Society, Proceedings of the 22nd Annual International Conference of the IEEE. Chicago, IL. Vol 4, 3064-3066, (2000).

8. Peter Ungar, Teeth Tell Ancient Tale, <http://www.physorg.com/news82911778.html>, November 16 (2006).

9. Carmi Shaim, et al., New Model of Internet Topology Using k-shell Decomposition, http://arxiv.org/abs/cs.NI/0607080, 17th July, (2006).

10. "Fractal," Encyclopedia Brittanica, Vol. 6, pp 618-619, (2004).

11. G. Julia, Memoire sur l'iteration des functions rationnelles, J. de Mathematiques Pures et Appliquees, 4, 47-245, Reprinted (with related texts) in Julia, pp. 121-139 (1968).

12. P. Fatou, Sur les equations founctionnelles, Bull. Societe Mathematique de France, **47**, 161-271, **48**, 33-94 and **48**, 208-314 (1919-1920).

13. ANOVA: Analysis Of Variance between groups, http://www.physics.csbsju.edu/stats/anova.html.