Paper ID #36557

Vibration Analysis Projects of Lumped-Parameter and Distributed-Parameter Systems

Shengyong Zhang (Associate Professor of Mechanical Engineering)

Shengyong Zhang (syzhang@pnw.edu) is an Associate Professor of Mechanical Engineering at the Purdue University Northwest. He has teaching and research interest in the areas of computer modeling and simulation, vibration and acoustics, automobile lightweight design, and engineering education.

Vibration Analysis Projects of Lumped-Parameter and Distributed-Parameter Systems

Department of Mechanical and Civil Engineering
College of Engineering and Sciences
Purdue University Northwest

Abstract

Knowledge about vibration is desired for mechanical engineers to analyze, measure and control the harmful effects on machine performance. The introductory engineering vibration course offered to mechanical engineering students covers the vibration analysis on both lumped-parameter systems (single- and multiple-degree-of-freedom systems) and distributed-parameter systems (bars, beams, etc.). This paper documents an effort of integrating two FEA-based projects in the mechanical vibration course. The first project makes use of quarter-car models to simulate the behavior of vehicle suspension systems. The main function of a suspension provides the necessary ride isolation and can be simulated by using a lumped-parameter system with mass, spring and damper elements. In this project, students build FEA models and simulate the vibration responses of a suspension to harmonic excitation in the frequency domain and to impulsive excitation in the time domain. The second project carries out the vibration analysis of an airplane wing which is simulated by using a distributed-parameter system. In this project, students build wing parametric CAD models and corresponding vibration modal characteristics of the wing models are investigated in the subsequent FEA-based vibration analysis. In general, student feedback on integrating the two vibration projects in the course is positive.

1. Introduction

Vibration is destructive in most mechanical systems and structures. Unbalance-induced blade vibration causes fatigue in turbines and can eventually lead to failure. Machining vibration in cutting tools can lead to a poor surface finish and reduce the life of cutting tools. The annoying oscillation as a car ride over a bumpy road conveys an impression of poor quality to the customers and the structure-borne cabin noise creates a nuisance to passengers. It is accepted that vibration in many cases is a limiting factor in machine designs and manufacturers set performance standards for their products to avoid excessive vibration [1].

Knowledge about vibration is desired for mechanical engineering students. The contents in this introductory engineering vibration course are built on previous courses in mechanical engineering curriculum. The prerequisites include the dynamics (for example, kinematic and dynamic analysis, principles of energy) and mathematics (for example, Laplace transform, eigenvalue problems). Course objectives include learning how to perform vibration analysis for lumped-parameter and distributed-parameter systems, design for vibration isolation and absorption, and conduct vibration experimental measurements. In a lumped-parameter vibrating system, all mass elements are considered as rigid bodies separated by spring and damper elements. Vibration analysis for lumped-parameter vibrating systems in this class includes calculating free and forced vibration responses of both single-DOF and multiple-DOF systems subjected to various excitations and calculating natural frequencies and vibration modes under specified boundary conditions. In a distributed-parameter system (rod, beam, etc.), the flexibility of components is considered. The mass and stiffness properties are modeled as being distributed throughout the spatial definition of the component. Vibration analysis for distributed-parameter vibrating systems in this class focuses on the modal analysis for natural frequencies and mode shapes.

Employing vibration analysis projects in teaching the engineering vibration course helps students develop critical thinking and problem-solving skills. Combined with hands-on projects [2, 3], FEA has been used as a supplement in the teaching of mechanical vibration for decades [4, 5]. Integrating appropriate FEA-based projects in teaching is an efficient way to assist students in the learning of engineering vibration. Animations from FEA enable students to visualizing the phenomena of vibrations and enhancing their comprehension of concepts and theories.

This paper documents an effort of integrating two FEA-based projects in the mechanical vibration course. The first project makes use of quarter-car models to simulate the behavior of vehicle suspension systems. The main function of a suspension provides the necessary ride isolation and can be simulated by using a lumped-parameter system with mass, spring and damper elements. In this project, students build FEA models and simulate the vibration responses of a suspension to harmonic excitation in the frequency domain and to impulsive excitation in the time domain. Furthermore, students investigate the variations of vibrating responses to changes in the parameters of shock absorbers. The second project carries out the vibration analysis of an airplane wing which is simulated by using a distributed-parameter system. In this project, students build wing parametric CAD models in which the wing length, root chord length and tip chord length are defined as design variables. Corresponding vibration modal characteristics of the wing models are investigated in the subsequent FEA-based vibration analysis.

2. Vibration analysis of lumped-parameter quarter-car models

Quarter-car model is used to simulate the behavior of vehicle suspensions. The main function of auto suspensions is usually simulated by spring and damper components which provide the necessary ride isolation at each wheel. Figure 1 shows a simple quarter-car model. The sprung mass (M) represents the mass of the vehicle supported on the suspension and the unsprung mass (m) is defined as the total mass of all parts being directly connected to the wheel. The stiffness and damping coefficient of the suspension in Figure 1 are represented by K_s and C_s, respectively. K_t is the tire stiffness. The damping effect of the tire is ignored compared to that of the suspension. Z_r is the excitation acting on the quarter-car model from the road surface, Z_u and Z are the vibration responses of the unsprung and sprung mass, respectively.

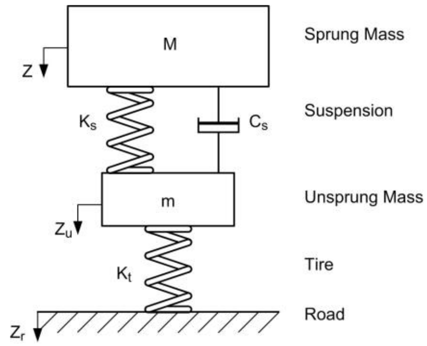


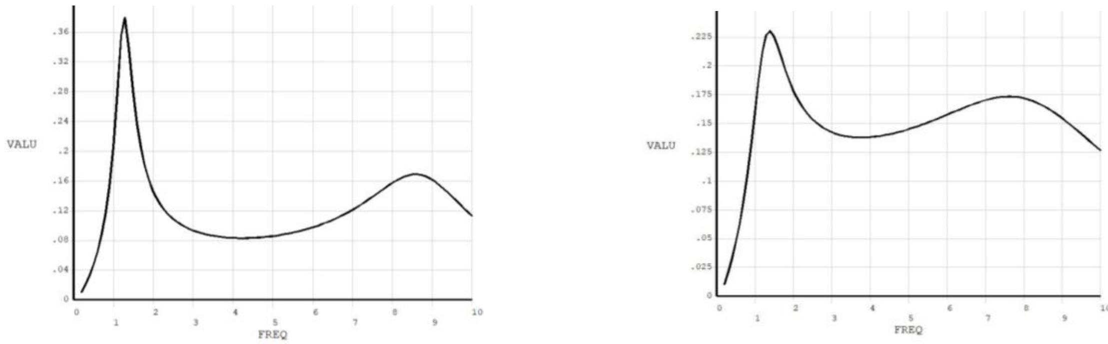
Figure 1. Schematic diagram of quarter-car model. M=450Kg, m=90kg, K_t=250000N/m, k_s=31250 N/m, and C_s=3000 or 1500 Ns/m.

2.1 Harmonic excitations

Harmonic excitation refers to a sinusoidal external excitation of a single frequency applied to the system. Harmonic excitations are a common source of external force. In addition, any arbitrary periodic force function can be expressed mathematically in terms of an infinite sum of harmonic functions (sines and cosines) using Fourier series. Knowing the vibration response to individual term in the Fourier series allows the total vibration response to be represented as the sum of the response to each individual harmonic term.

In this project the road excitation (Z_r) is chosen to be the harmonic input to the suspension, and it will be transmitted to the occupants through the wheel/tire assembly and suspension system. From the ride quality point of view, designers are mostly interested in the vibration of the sprung mass, which is used as the output in this project. Figure 2 shows the vibration responses of the quarter-car model (as shown in Figure 1) in a form of displacement to harmonic excitation with driving frequencies up to 10 Hz. Resonance is a critical concept related to forced vibration and it occurs when a periodic external force is applied to a system having one or more natural frequencies equal to the driving frequency. The resulting vibration response will be amplified and can cause very large displacement. The two peaks in Figures 2(a) and 2(b) correspond approximately to the two natural frequencies of the quarter-car model.

Damping restrains vibratory motion by dissipating energy and is of great importance in limiting the amplitude of vibration at resonance. Students understand that the steady-state response to a harmonic excitation is a function of driving frequency and damping coefficient. Theoretically, increasing damping will help reduce the magnitude of the vibration response, especially at resonance frequencies. In this project, students are instructed to change damping coefficient of dashpots to validate the damping effect on reducing the vibration response magnitude. Figure 2(b) shows the vibration responses of the quarter-car model with increased damping coefficient. It clearly illustrates the effect of damping on response magnitudes - the response magnitude near to the natural frequency is reduced significantly with increasing damping ratio. The negative effect of increasing damping will be discussed in the following impulsive response analysis.

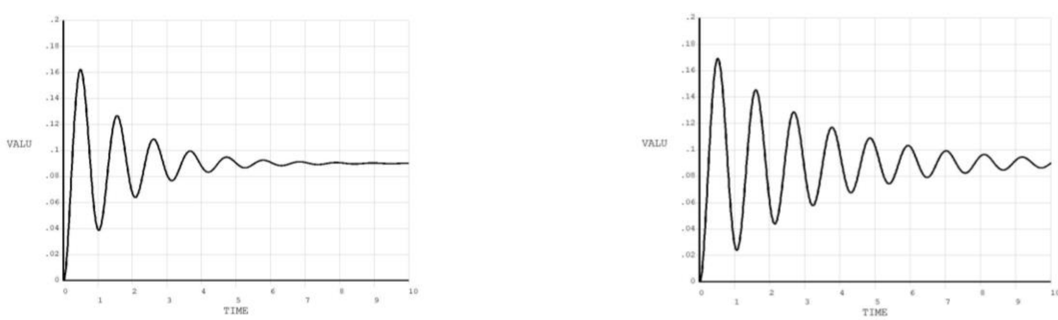


(a) (b)

Figure 2 Responses of the sprung mass to harmonic excitations in the frequency domain. (a) C_s=1500Ns/m and (b) C_s=3000Ns/m

2.2 Impulse excitations

An impulse excitation is a force that is applied for a very short length time, such as the force acting on a car when running over a speed bumper. The impulse of a force is defined to be the integral of the force over the time interval for which it acts and provides a measure of the strength of an applied impulsive force. Impulse excitations are a common source of external force. In addition, a general force excitation can be considered as being constituted of infinitesimal impulses. Knowing the vibration response to individual impulse allows the total response to be represented as the sum of the response to each individual impulse based on Duhamel integral by using Laplace transform and inverse Laplace transform. FEA modelling of the quarter-car model provides students a visual insight into the above mathematical results. Figure 3 shows the transient impulsive responses of the quarter-car model with different damping coefficient (C_s) in the suspension shock absorbers. By comparing the transient responses in Figures 3(a) and 3(b), students conclude that increasing damping will increase the setting time.



(a) (b)

Figure 3. Responses of the sprung mass to impulsive excitations in the time domain. (a) C_s=1500Ns/m and (b) C_s=3000Ns/m.

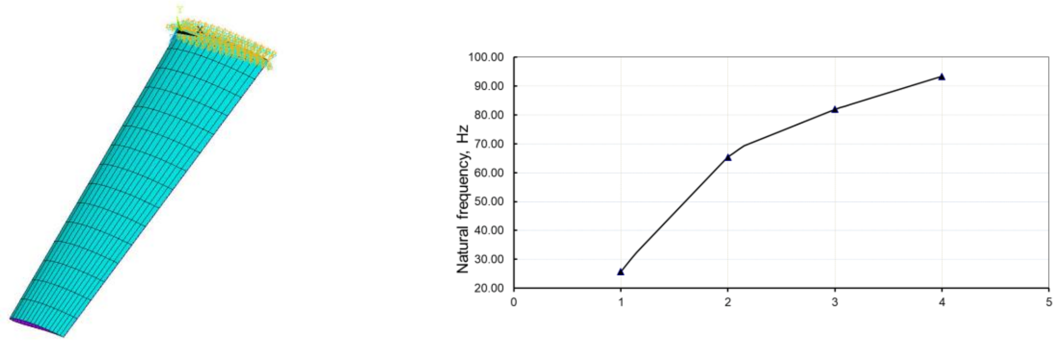
This project embodies fundamental concepts and theories covered in the vibration analysis for lumped-parameter vibrating systems, such as MDOF systems, forced responses, impulsive responses, design for vibration depression, etc. It contributes to the following course objectives, including

- Being able to conduct free and forced responses of damped and undamped, single- and multi-degree-of-freedom systems.
- Understanding vibration isolation and absorption.
- Being able to perform vibration analysis using FEA tools.

3. **Vibration analysis of distributed-parameter aircraft wing models**

In moving from lumped-parameter systems to distributed-parameter systems, the mass of an object is distributed throughout the structure as a series of infinitely small mass elements. The infinite number of elements move relative to each other in a continuous fashion when the object is in vibration. A distributed-parameter system has an infinite number of DOFs and hence an infinite number of vibration modes and corresponding natural frequencies. The detailed analytical vibration analysis of a string with fixed ends is discussed in class. First, the partial differential equation (PDE) is built based on the free body diagram of an infinitesimal element from the string. In the PDE, the lateral motion at any point on the string is expressed as a function of the time and position along the string. Secondly, the PDE is expressed as a system of two ordinary differential equations (ODE) using the separation of variables method for analytical solution. Finally, the two ODEs are solved with specified boundary conditions for the string motion, from which the vibration modes and natural frequencies of the given string can be derived. Analytical analysis for vibration responses of even the simplest vibration case is a challenge to many engineering students, partially because of the advanced mathematics involved in this process. In addition, there are only a few distributed-parameter models that have analytical solutions. Most realistic engineering problems cannot be solved using analytical methods. Fortunately, FEA has unique abilities in simulating the performance of a mechanical part or system and has been employed to solve problems relating to engineering vibrations by providing alternative numerical solutions.

Aircraft wings generate most of lifting force and provide the required stability during flight. In its most basic form, a wing is comprised of spars (extending from the fuselage to the wing tip), ribs (spanning between the leading edge to the trailing edge), and the outside skin panels (covering the entire surface of the wing) [6]. Vibration is a crucial limiting consideration in the analysis and design of aircraft wing structures to avoid disastrous failures due to the propagation of existing cracks in the material. The outside skin panels have been simulated in this project. The modelling process begins by preparing the airfoil profile coordinates in a Microsoft Excel spreadsheet. Parametric CAD models are developed to capture design intent with defined design variables, including the wing length and chord length at the root and tip of the wing (effectively defining the taper ratio). With the completed CAD models, students continue performing subsequent FEA-based vibration modal analysis to study the vibration properties. Modal analysis determines the inherent natural frequencies and mode shapes of wing models. A mode shape describes the vibration pattern of the wing when vibrating at each natural frequency. Figure 4(a) shows the FEA model in which all the degrees of freedom at the root end are rigidly constrained. Without losing generality, Figure 4(b) plots the first four natural frequencies obtained from modal analysis.



(a) (b)

Figure 4. (a) Wing FEA model with rigid constraints applied at the root end, and (b) first four natural frequencies of the wing obtained from modal analysis.

Combined with the analytical vibration analysis of a string in class, this project employs FEA to conduct the modal analysis for numerical solutions of a distributed-parameter vibration system that cannot be solved analytically. It contributes to the following course objectives, including

- Being able to conduct longitudinal, torsional, and lateral vibrations of continuous systems.
- Being able to perform vibration analysis using FEA tools.

Both projects are assigned as group projects for enhancing collaboration among students and making decisions efficiently. Each group has three to four students and is teamed up without interference from the instructor. Students are required to collect their own data. For example, each team choose the vehicle model on which you are going to investigate, search for the specifications of the selected vehicle, and determine all required data (sprung mass, unsprung mass, stiffness and damping coefficient of suspension, and stiffness of tire) for the quarter-car model project. Subsequent FEA models are developed in ANSYS with the assistance of instructor. FEA results (i.e., harmonic responses and impulsive responses in the quarter-car project, vibration modes and natural frequencies in the airplane wing project) are included in the project reports, along with the project objectives and concluding remarks. Grading of reports is based on the sufficiency of analysis and completeness of report. Each student is evaluated by other students, including teammates, in the oral presentation. Grading of oral presentation is based on the logical sequence of presentation and responses to audience questions. Most students are good at CAD modelling. FEA modelling, however, is a challenge to most students in these projects. One main reason is that FEA class is designed to be an elective course in the program curriculum and students has few experiences with FEA techniques.

The author understand it is imperative to gather the student perspective for assessing the effort of introducing the FEA-based projects in the engineering vibration course. Unfortunately, there is not enough data from students so far for comprehensive assessment of the effort. One informal feedback from students is that the projects provide a visually oriented insight into engineering vibrations. Animations from FEA enable students to visualize the phenomena of vibrations, enhancing their comprehension and grasp of concepts (vibration resonance, vibration mode) and theories (vibration responses to harmonic and impulse excitations, modal analysis).

Another feedback from students indicates that more realistic engineering problems would be beneficial to foster critical thinking and promote professional development.

4. Conclusions

This paper documents an effect of integrating two FEA-based projects as a supplement in teaching the engineering vibration course. The first project is based on lumped-parameter quarter-car models. Students investigate the vibration responses of vehicle suspensions subjected to harmonic and impulse excitations. The second project is based on distributed-parameter aircraft wing models. Students build parametric wing CAD models and subsequently perform FEA vibration analysis for natural frequencies and modes. Informal student feedback on integrating the two projects is positive.

Acknowledgements

This research is sponsored through the Indiana Space Grant Consortium (INSGC). The support is gratefully acknowledged.

References

[1] D.L. Inman, *Engineering Vibration*, 4rd edition, Prentice Hall, 2014.
[2] R. J. Ruhala, "Five Forced-Vibration Laboratory Experiments using two Lumped Mass Apparatuses with Research Caliber Accelerometers and Analyzer," *ASEE Annual Conference and Exposition*, Vancouver, B.C. Canada, June 26-29, 2011.
[3] C. C. McDaniel and G. C. Archer, "Full-scale Mechanical Vibrations Laboratory," *ASEE Annual Conference and Exposition*, Atlanta, Georgia, June 23-26, 2013.
[4] J. R. Baker, "MATLAB-Based Finite Element Analysis in a Vibrations Class," *ASEE Annual Conference and Exposition*, Indianapolis, Indiana, June 15-18, 2014.
[5] S. Zhang, and A. Togbe, "Engineering Application Projects for Teaching Engineering Mathematics and Numerical Methods," *127th ASEE Annual Conference & Exposition*, Virtual Conference, June 22-26, 2020.
[6] S. Zhang, and M. Mikulich, "Parametric CAD Modelling of Aircraft Wings for FEA Vibration Analysis," *Journal of Applied Mathematics and Physics*, 9, pp. 889-900, 2021.

Biographical Notes

Dr. Shengyong Zhang (syzhang@pnw.edu) is an Associate Professor of Mechanical Engineering at the Purdue University Northwest. He has teaching and research interest in the areas of computer modeling and simulation, vibration and acoustics, automobile lightweight design, and engineering education.