

## **AC 2008-65: VIBRATIONS LABS TO HELP ACHIEVE A RESONANCE IN LEARNING**

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## Vibration Labs to Help Achieve a Resonance in Learning

### Abstract

A sequence of laboratories has been designed and implemented in an undergraduate course in mechanical vibrations to introduce students to experimental aspects of vibrations and experimental modal analysis. Unfortunately, undergraduate vibration courses, especially if they do not have a lab associated with them, are often perceived by many students to be courses in differential equations. By exposing students to vibration measurement instrumentation such as accelerometers and dynamic signal analyzers, and by allowing them to take experimental data, calculate frequency response functions, and identify system parameters and mode shapes, student learning and motivation is enhanced. One characteristic of the labs described in this paper, in contrast to other vibrations labs discussed in the literature, is the way each lab builds upon the previous one and the fact students test real engineering structures. The initial labs in the course use Electronic Control Products (ECP) hardware and introduce the idea of frequency response functions (FRFs) and system identification. After students are familiar with these ideas, they progress to using PHOTON II's (a 32 bit, 4 channel data acquisition system), RT Pro for data acquisition and signal processing, and DIAMOND for system identification and mode shape animation. In this paper the labs will be described and assessment results presented as to their efficacy.

### Introduction

According to the dictionary, resonance is “a vibration of large amplitude in a mechanical or electrical system caused by a relatively small periodic stimulus of the same or nearly the same period as the natural vibration period of the system.”<sup>1</sup> Thus, a large motion can result from a very small stimulus if it is at just the right frequency, that is, at the natural frequency of the system. Is there an analogous phenomenon in learning? Do some teaching strategies result in a larger amount of learning, or a “resonance” in learning, than others? How does one achieve this resonance?

According to the National Research Council report *How People Learn: Brain, Mind, Experience and School*<sup>2</sup>, one aspect of effective learning is its durability, that is, does the learning have long-term impact in the ways it influences other kinds of learning or performance? A key conclusion in this report on the concept of durability is the fact that it is essential for a learner to develop a sense of when what has been learned can be used--the conditions of application. Failure to transfer is often due to learners' lack of this type of conditional knowledge. If the problems examined in a vibrations course are always presented in the context of idealized mass, stiffness and damping elements, i.e. looking nothing like a realistic system, then it will be difficult for students to apply the concepts discussed in the course.

Unfortunately, vibrations courses often do not have a dedicated lab associated with them, and the only exposure students have to vibrations experiments are in more general laboratory courses on

engineering measurements<sup>3-4</sup>. These labs typically consider a relatively simple structure, such as a cantilevered beam.

There do exist, although they are rare, vibrations and structural dynamics courses with integrated lectures and laboratories<sup>5-8</sup>. Often the purpose of these labs is to illustrate the main concepts in the lecture with hands-on laboratory exercises. This approach can help motivate the course material, but also allows students to see differences between theory and measured experimental results<sup>5-6</sup>. This is critical in helping students understand that models have assumptions and limitations. Although this laboratory philosophy introduces students to experimental approaches using transducers and data acquisition systems, it does not necessarily prepare them to design and execute a vibration experiment of a realistic structure.

At the University of Toledo the laboratory experiments for a Mechanics and Vibration Laboratory have been redesigned to transform the learning process from a subject-based learning to a problem-solving learning<sup>7</sup>. This course has Mechanical Vibrations as a prerequisite, and therefore the lab does not enhance the vibrations course. One goal for this course is to provide students with more hands-on experiences and to challenge them by requiring the procedure for each laboratory experiment to be designed and carried out by each group of students. The authors state that due to the number of students and the limited number of lab sessions, it was difficult to provide the students with the real hands-on experience with the instrumentation and lab setup as desired.

An innovative new approach for integrating vibrations lectures and lab material is a course, ME 497A “Practical Experiences in Vibration,” which was first offered at Purdue in 2003<sup>9</sup>. This course involves a “roving laboratory” for undergraduate students. Experiments in the roving laboratory are to be carried out in class, in two different on-campus facilities, and in the field. Experiments are used by the instructor to motivate each and every theoretical discussion in class, to teach students how to plan, conduct and interpret their own experiments, and to expose students to important emerging areas of experimental mechanics. Students are then required to design and implement a vibration test. The original offering of this course only had 14 students, and it is not clear if this approach is scalable to larger classes.

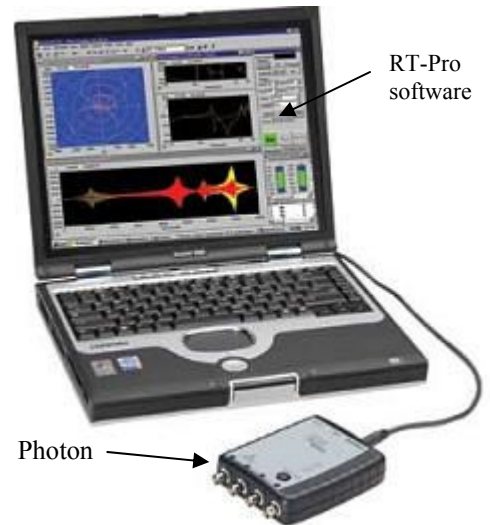
It is clear that experimental vibration analysis is an important tool for identifying dynamic properties of mechanical structures, and yet this important tool is not a part of many vibration courses. Experimentation is also critical to helping students understand that models and reality are always different, although some models are better than others. Poincare stated it this way: “experiment is the sole source of truth.”<sup>10</sup> To help bridge this gap between experimentation and theory, and to equip students with practical skills in experimental vibrations, a lab has been added to the Mechanical Vibrations course at Rose-Hulman. The labs are designed to build upon those associated with prerequisite courses, and each week’s laboratory experience builds upon the previous week’s. The ultimate object of the lab is for the students to design and conduct a modal test of a structure of their choice.

### **Laboratories in Vibrations**

Starting in the 2006-2007 academic year, Vibrations at Rose-Hulman included a scheduled once a week laboratory. This reduced the number of lecture hours from 40 to 30, but there was now

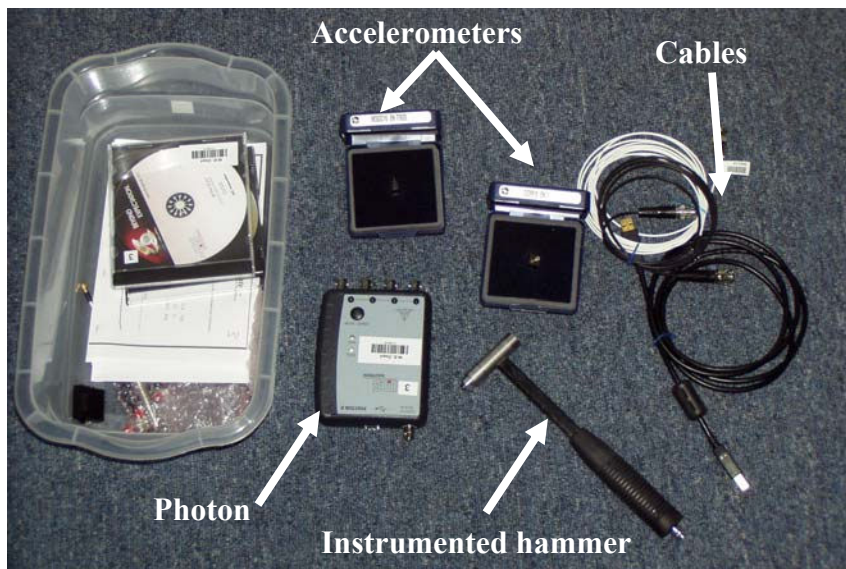
time available for ten 3-hour labs. The labs in Vibrations were designed to build on several labs that were performed in previous courses. These previous labs included topics such as using Simulink, system identification, and frequency response of systems. These previous labs have been discussed in detail in Reference 11. For the 2007-2008 academic year new labs have been created because the school acquired 10 LDS PHOTON II's as shown in Figure 1. These are currently being integrated into the laboratory. These small portable data acquisition systems have the following characteristics:

- Multi-channel: 2 to 4 inputs; 1 output; tachometer
- Real-time analyzer: 56 KHz rate for all channels
- 32-bit processing: 32-bit floating point DSP
- Wide dynamic range: 24-bit inputs and output; 110 dBfs
- Built-in conditioning: ICP sensor power
- Easy installation: USB interface for hot "Plug & Play"
- Easy to use: Windows XP/2000 interface tailored for noise and vibration testing
- Light and rugged: 8 ounces (227g)



**Figure 1.** LDS PHOTON II

In addition to the PHOTONs, instrumented hammers and accelerometers have also been acquired for the labs. A total of 10 "kits" were created to accommodate 10 lab groups. Each kit consists of a PHOTON, an instrumented hammer, a 10 mV/g accelerometer, and a 100 mV/g accelerometer. A photo of a kit is shown in Figure 2. The total cost of each kit is approximately \$7,000 for all the hardware and software, but a similar kit could probably be assembled using less expensive hardware.



**Figure 2.** Vibrations kits used in lab

Brief descriptions of the current laboratories are shown in Table 1. From Table 1 it can be seen that these laboratories can be grouped into four different categories: Introductory lab, ECP labs, PHOTON and DIAMOND labs, and project work time.

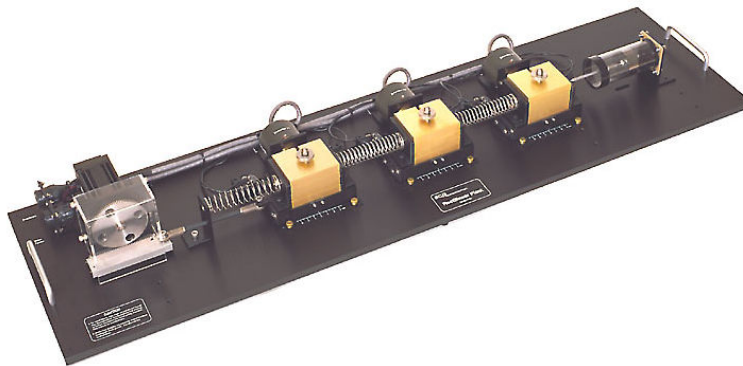
Table 1 – Description of laboratory activities in Mechanical Vibrations	
Week	Brief description of lab activities
1	<ul style="list-style-type: none"> <li>• Load RT Pro software on laptops (for data acquisition and signal processing)</li> <li>• Load DIAMOND<sup>12-13</sup> (for curve fitting and mode shape animation)</li> <li>• Introduce the lab and project</li> <li>• Review labs from previous courses</li> </ul>
2	<ul style="list-style-type: none"> <li>• Frequency response of a SDOF system (obtained using one frequency at a time and by using a swept sin) using the ECP system<sup>14-15</sup></li> <li>• FRF calculation of swept sin data using Matlab command tfestimate</li> <li>• System identification using peak picking</li> </ul>
3	<ul style="list-style-type: none"> <li>• Mini-lecture on tfestimate (to determine FRFs) and fminsearch (to identify system parameters)</li> <li>• System identification for data from week two</li> <li>• Take data and compare FRFs found using a swept sin input and a band-limited random input</li> </ul>
4	<ul style="list-style-type: none"> <li>• Prelab using a Working Model simulation a 2-DOF system <ul style="list-style-type: none"> <li>◦ Study what happens when passing through a resonance at different speeds</li> </ul> </li> <li>• Swept sine data for 2-DOF ECP system</li> <li>• Frequency and damping for each peak using fminsearch (SDOF fits) and using a peak pick and half-power points</li> </ul>
5	<ul style="list-style-type: none"> <li>• Prelab to watch steaming video of a modal test by Pete Avitabile from UMass, Lowell and to read Reference 16 on experimental modal analysis</li> <li>• Discuss Ref. 16</li> <li>• Mini-lecture on the difference between displacement FRFs and acceleration FRFs and on instrumentation and accelerometers</li> <li>• Demonstration of how to perform data acquisition and transfer the data to DIAMOND</li> <li>• Provide trial geometry and data to load into DIAMOND to analyze</li> </ul>
6	<ul style="list-style-type: none"> <li>• Perform modal test on a plate <ul style="list-style-type: none"> <li>◦ Take data using the PHOTON, export to DIAMOND, and use DIAMOND for curve fitting and mode shape animation</li> </ul> </li> </ul>
7	Begin project to perform a modal test on a structure of their choice
8	Work on project
9	Work on project, discuss oral presentation
10	Present modal test results

### Introductory labs

The 2007-2008 academic year was the first time the PHOTONS were used, so it was unclear how long it would take to install the software on all the students' laptops. For this reason, the first lab was intended to provide an overview of what to expect in the lab, to load the appropriate software onto the students' laptops, and to introduce the experimental modal analysis project. The first piece of software students needed to load onto their laptops was RT Pro. RT Pro is a powerful program for data acquisition and real-time signal processing for dynamic signal analyzers. The second piece of software was DIAMOND. This software was developed at Los Alamos National Laboratory, and it is a MATLAB software toolbox which implements several state-of-the-art techniques for experimental modal analysis, damage identification, and finite element model correlation and refinement in a menu and button-driven graphical user interface<sup>12-13</sup>. Only the experimental modal analysis capabilities are used in this course. The first lab also contained a review of the labs in previous courses that were directly applicable to this course.

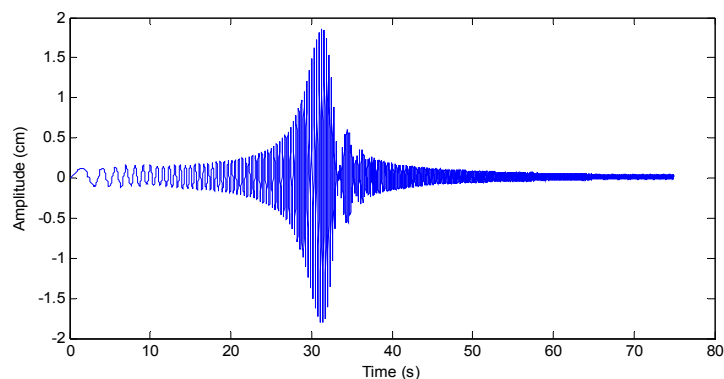
### ECP Labs

The second group of labs, Labs 2 – 4, used the Educational Control Products (ECP) Rectilinear Control System<sup>14</sup>, as shown in Figure 3. This hardware has been used for vibration labs at other universities<sup>8</sup> and at Rose-Hulman<sup>15</sup>. Rose-Hulman has 12 of these systems, allowing students to work in teams of two or three. The ECP rectilinear system is a translational mass-spring-damper system driven by a DC electric motor that provides up to three degrees of freedom of motion. The springs between carts may be changed to the user's liking. A variable air damper may be connected to any of the masses. The plant also provides for varying the system mass by adding or removing masses.



**Figure 3.** The ECP Rectilinear Control System

Lab 2 required students to generate a FRF for a SDOF system by inputting one frequency at a time and by using a swept sine input. Typical data for a swept sine input to the SDOF system are shown in Figure 4. From the swept sine



**Figure 4.** Typical time response for a swept sin input

results the FRF can be calculated in Matlab using the built-in command called “tfestimate.” A typical FRF generated this way is shown in Figure 5.

This lab was also used to begin introducing students to methods of system identification. In experimental modal analysis, the process of obtaining the natural frequency,  $\omega_n$ , and damping ratio,  $\zeta$ , is called “curve fitting.” The first method introduced for estimating  $\omega_n$  and  $\zeta$  was the “peak picking” method. If the magnitude plot looks like the curve shown in Figure 5, that is, it has small damping, then the damping ratio can be estimated from the half-power points as shown in Eq. 1.

$$\zeta_i \approx \frac{\omega_2 - \omega_1}{2\omega_n} \quad (1)$$

where  $\omega_1$  and  $\omega_2$  are the frequencies associated with the half-power points as shown in Figure 5.

Lab 3 began with a short lecture on using tfestimate. Students used this command in Lab 2, but without any explanation of what it was doing. They now had an appreciation for what it can be used for, and this lecture provided some background as to how Matlab is calculating the FRFs and the effect of the various input parameters on this function. The Matlab command is shown in Eq. 2

$$[Txy, F] = \text{tfestimate}(f, x, \text{window}, \text{noverlap}, \text{nfft}, \text{sample\_rate}) \quad (2)$$

where

Txy: the vector containing the complex values of the FRF at each frequency point

F: the vector of frequencies

f: the input force time history

x: the output time history

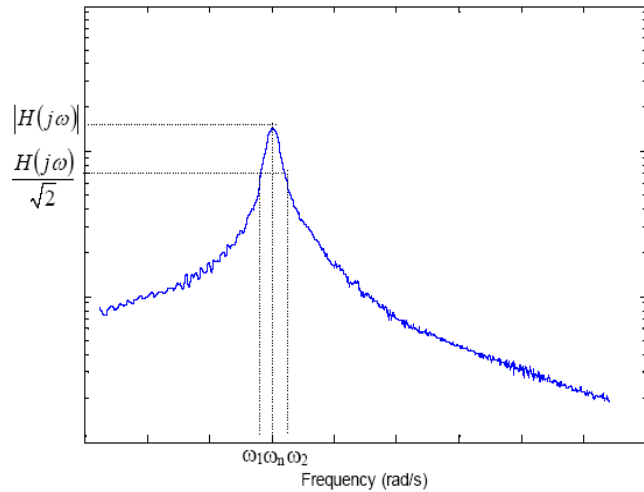
window(N): a window of length N data points that is to be applied to the data

noverlap: the amount of overlap between sections

nfft: the number of FFT points to be used in the calculation

sample\_rate: the sample rate used for the data.

The overlap, the number of points in the FFT and the total length of the input vectors determine the amount of averaging that is done in calculating the FRFs. Since averaging does not make a lot of sense for a swept sine input, students were required to also use a band-limited random input for this lab. Using the band-limited random data allowed students to investigate the effect of windowing, averaging and the number of points used on the resulting FRFs.



**Figure 5.** FRF generated using tfestimate and the swept sin data

Lab 3 also began to introduce students to more complex curve fitting concepts. Rather than using the half-power points, the built-in Matlab command called “fminsearch” was used to minimize the difference between the experimental FRF and an analytical model using a performance index given by Eq. 3.

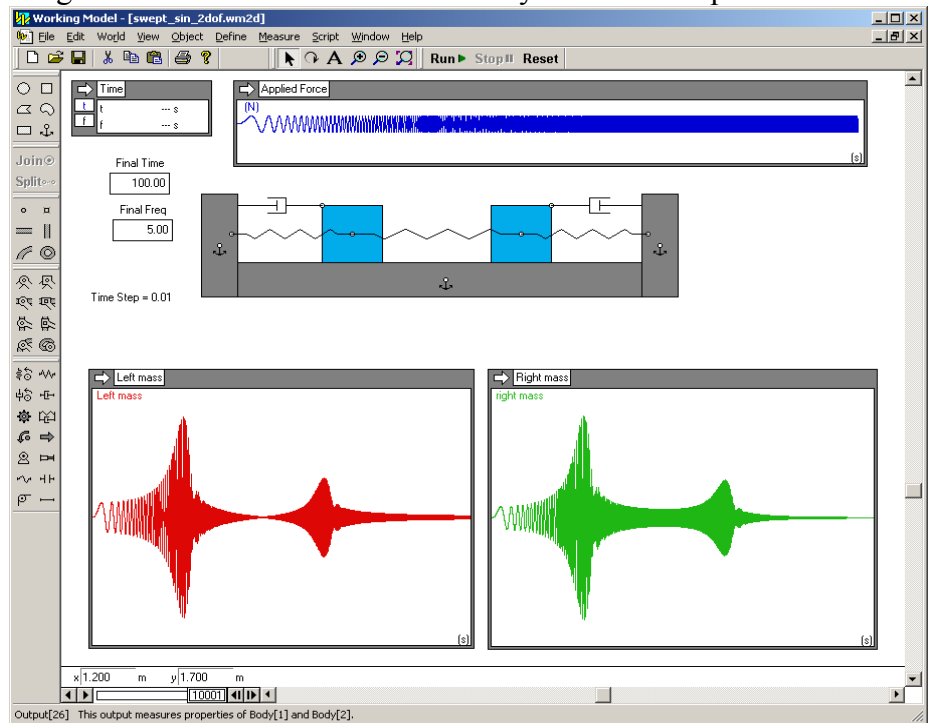
$$J = \frac{1}{n} \sum_{i=1}^n \left( |H(j\omega_i)|_{\text{model}} - |H(j\omega_i)|_{\text{exp}} \right)^2 \quad (3)$$

where  $|H(j\omega_i)|_{\text{model}}$  is the magnitude of the FRF at the  $i^{\text{th}}$  frequency from a theoretical model and  $|H(j\omega_i)|_{\text{exp}}$  is the experimental value of the FRF at the  $i^{\text{th}}$  frequency. The theoretical model used for this lab is given by Eq. 4

$$|H(j\omega)| = \frac{K}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad (4)$$

where  $K$  is the static gain,  $r$  is the frequency ratio and  $\zeta$  is the damping ratio. The values of  $K$ ,  $\omega_n$ , and  $\zeta$  were identified using `fminsearch` by minimizing a performance index similar to the one shown in Eq. 3. Students were required to do this minimization using the magnitude in linear units and also in dB.

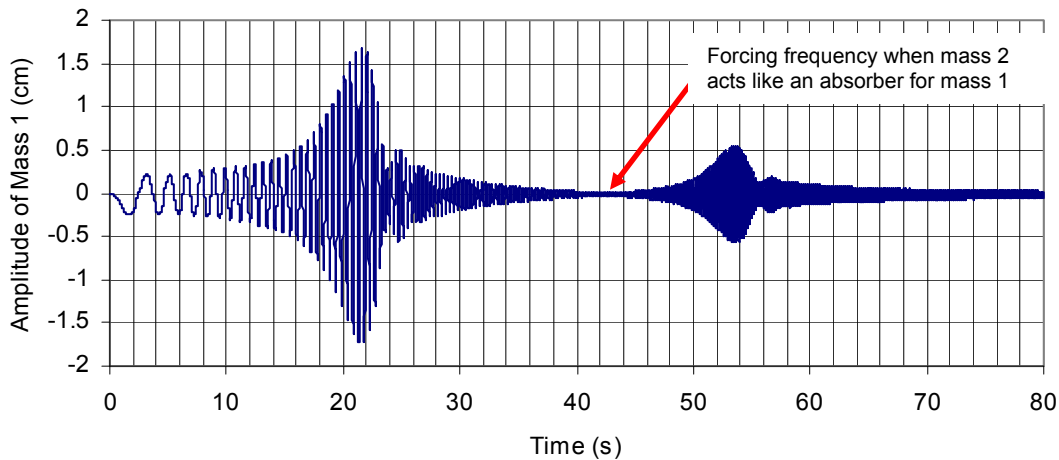
Lab 4 was similar to Lab 3 except a 2-DOF system was examined. A prelab for this lab involved students investigating a Working Model simulation of a 2-DOF system. A snapshot of this simulation is shown in Figure 6. In this simulation a swept sine is applied to the mass on the left and the displacement of the two masses are plotted. This simulation allows students to investigate the effect of using different rates for the swept sine, both to observe when the mass on the right acts like a vibration absorber for the mass on the left, and to give students an idea of what to expect when taking data in lab.



**Figure 6.** Snapshot of a Working Model simulation similar to the 2-DOF system examined in Lab 4.



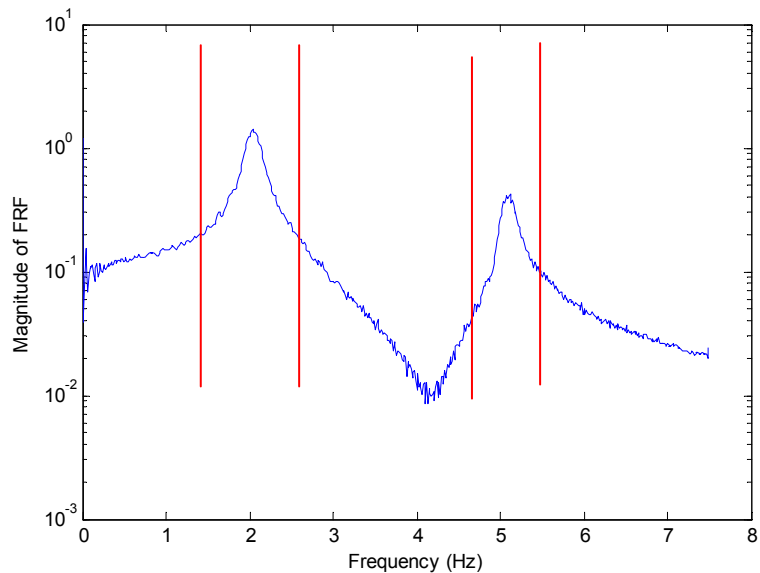
Experimental swept sine data for a 2-DOF system is shown in Figure 7. Clearly the experimental data in Figure 7 is similar to what the students observed in the prelab. From this experiment students made observations about the two modes of vibration. For example, at the first resonant frequency both masses had a large amplitude of vibration and were in phase. At the second resonance the two masses were 180 degrees out of phase of each other. It was also possible for them to notice when the second mass acted like a vibration absorber for the first mass so that the first mass had an amplitude very close to zero. This frequency is labeled in Figure 7.



**Figure 7.** Swept sine results.

A typical FRF for the mass with forcing is shown in Figure 8. This FRF was determined using Matlab. Using this data students perform a SDOF fit for each peak, thereby estimating the two natural frequencies and modal damping ratios for this system. Students were required to select a frequency band for each resonance as shown in Figure 8.

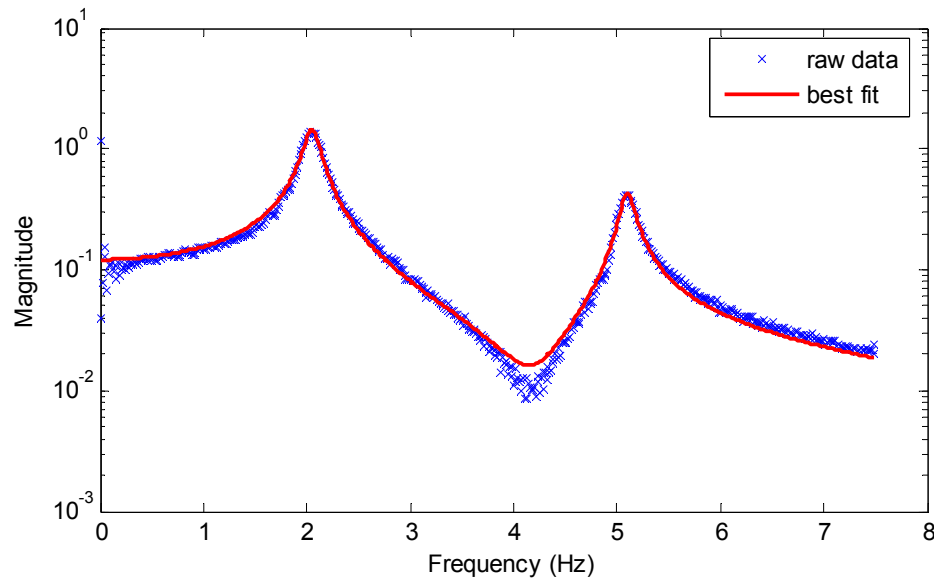
The concept of a MDOF curve fit was also introduced by using `fminsearch` and a 2-DOF model. Students used the transfer function shown in Eq. 5.



**Figure 8.** Typical FRF for a 2-DOF system and frequency bands used for curve fitting.

$$G(s) = \frac{K_1 + K_2s + K_3s^2}{(s^2 + 2\zeta_1\omega_1s + \omega_1^2)(s^2 + 2\zeta_2\omega_2s + \omega_2^2)} \quad (5)$$

A typical 2-DOF fit using `fminsearch` is shown in Figure 9. Although `fminsearch` is very convenient to use since it is a built-in Matlab command, it is not optimized for this type of problem, so it is very inefficient. More efficient curve-fitting algorithms exist and were introduced in labs 5 and 6.



**Figure 9.** Typical FRF magnitude plot and a best fit curve for mass 1.

### PHOTON and DIAMOND labs

The final set of labs involved using the PHOTON and having students perform tests on a real structure using a modal hammer, accelerometers, a data acquisition system, and the experimental modal analysis software, DIAMOND. In lab 5 experimental modal analysis was discussed. The prelab required students to listen to streaming videos of Pete Avitabile from UMass, Lowell demonstrating the use of the PHOTON and performing a modal test. Students were also required to read a good non-mathematical introduction to experimental modal analysis given in Ref. 16. When students came to lab the paper was discussed, and then there was a demonstration of how to use RT Pro and DIAMOND. A snapshot of typical data students would see using RT-Pro is shown in Figure 10. Trial geometry and data was provided to students to allow them to identify frequencies, damping and mode shapes using DIAMOND. The two curve fitting methods used in DIAMOND were the peak-pick method (similar to what the students did with the 2-DOF ECP system) and the Rational Polynomial Method where the students were required to select a frequency band and a number of modes in the band. A snapshot from DIAMOND is shown in Figure 11. In lab 5 we also discussed other aspects of modal testing such as linearity tests and reciprocity tests. The final topic discussed, which is common in any sort of testing and analysis, was how to export the data from one piece of software and then import it into another. In our case we were exporting the FRF data from RT Pro, and importing it into DIAMOND. This process required the students to concatenate the universal files from the RT Pro into a single file for DIAMOND.

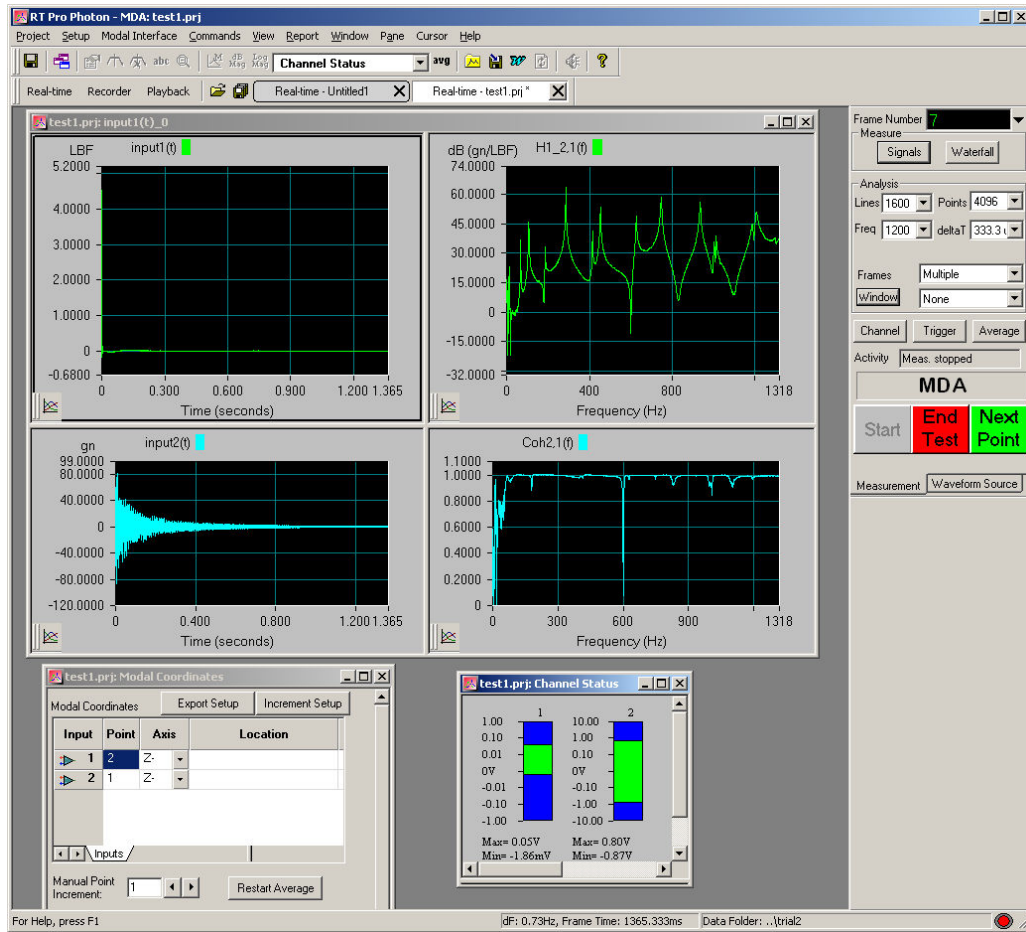


Figure 10. Snapshot from RT Pro

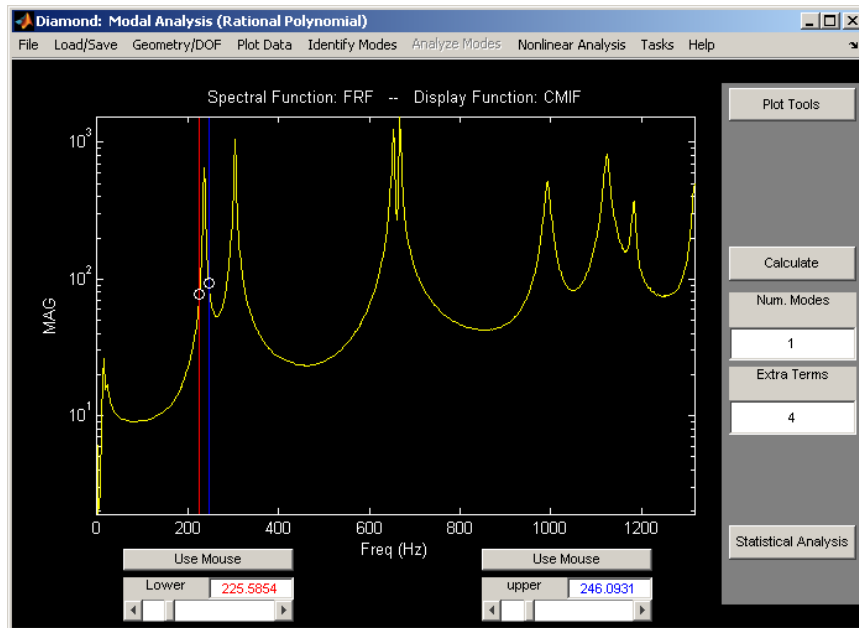


Figure 11. Rational polynomial window in DIAMOND

In Lab 6 students were required to perform a modal test on a plate to ensure that they knew how to perform a complete modal test. In this lab they selected test locations, the frequency range, number of averages, and other test parameters. After taking the data using the PHOTON and RT-Pro, they exported the data to DIAMOND and subsequently analyzed it in DIAMOND.

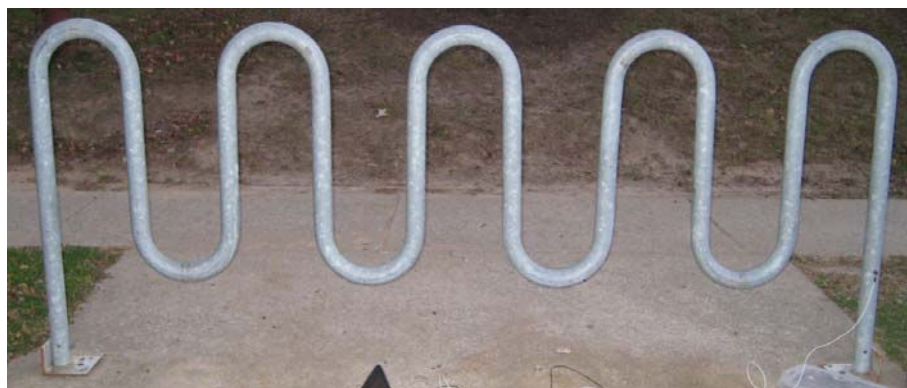
After mastering the use of the PHOTON and DIAMOND, students performed a modal test on a structure of their choice during weeks 7 through 9. The results from these tests were presented during the lab for week 10. Structures tested during the first time this course was offered included a modal test of a bicycle wheel (Figure 12), a guitar (Figure 13), a bike rack (Figure 14), a basketball rim, a lawnmower, a cymbal, a mountain bike frame, and many other structures.



**Figure 12.** Bicycle wheel that was tested



**Figure 13.** Guitar that was tested



**Figure 14.** Bike rack that was tested

## Assessment

To assess the laboratory, students were asked to rate their degree of agreement with a number of statements. A “1” indicated strong disagreement and “5” strong agreement. The results are shown in Table 2. A total of 44 students completed the assessment.

<b>Table 2 – Assessment results (N=44)</b>	
<b>Statement</b>	<b>Rating <math>\bar{x} \pm \sigma</math></b>
1. The lab in this course has helped motivate the topic of vibrations.	4.0 ± 0.7
2. The lab helped me to understand the theory better.	3.9 ± 0.7
3. As a result of this lab I have a much better understanding of experimental modal analysis than I did before the course.	4.4 ± 0.6
4. As a result of this lab I feel confident in performing a modal test of a structure.	4.1 ± 0.6
5. As a result of the lab my understanding of vibrations terminology (FRF, curve fitting, coherence, ICP, natural frequency, natural modes, etc.) has improved greatly.	4.2 ± 0.5
6. Overall rating of the vibrations lab this quarter. (1 – terrible, 5 – excellent)	4.0 ± 0.5

From Table 2 it is clear that student response was very positive and students felt that they gained a good understanding of experimental modal analysis as a result of the laboratory. Students were also asked to comment on “Good aspects of the lab” and to make “Suggested improvements to the lab”. The good aspects of the class most commonly mentioned were 1) the project, 2) the “real world” and “hands on” nature of the labs and 3) the excellent experimental equipment. The most common suggested improvements were 1) to reduce the workload, and 2) to try and reduce the hardware and software problems. Some students also did not enjoy the ECP labs as much as the project, but they did not have suggested improvements. This is the first class I’ve ever taught where I had numerous students say “great project!” or “I really enjoyed the lab.” One student commented, “The lab was really beneficial as I talked to employers. One company was very interested in my abilities in vibration analysis and testing.” One suggestion, which will be implemented next time I teach the course, is to have a project progress report where students discuss their experimental set-up with me and present preliminary results.

## Conclusions

A lab has been developed for a senior level mechanical vibrations course. This lab was designed to help bridge the gap between experimentation and theory and to equip students with practical skills in experimental vibrations. The labs were designed to build upon those associated with

prerequisite courses, and each week's laboratory experience builds upon the previous week's. By the end of the course students have acquired skills in using data acquisition hardware and software, dynamic signal analyzers, and experimental modal analysis software. The ultimate objective of the lab was for students to design and conduct a modal test of a structure of their choice, which was successfully accomplished. Student response to the new lab has been very positive.

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