Visual Differential Calculus

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***Abstract*— This expository paper is intended to provide engineering and technology students with a purely visual and intuitive approach to differential calculus. The plan is that students who see intuitively the benefits of the strategies of calculus will be encouraged to master the algebraic form changing techniques such as solving, factoring and completing the square. Differential calculus will be treated as a continuation of the study of branches[11] of continuous and smooth curves described by equations which was initiated in a pre-calculus or advanced algebra course. Functions are defined as the single valued expressions which describe the branches of the curves. Derivatives are secondary functions derived from the just mentioned functions in order to obtain the slopes of the lines tangent to the curves. The derivatives of the derivatives are related to the turning of the tangent lines. The concepts involved in the study of continuous curves are not difficult to comprehend. But parsing complexity arises from the many combinations of forms, kinds and operations on functions. The paper will focus on the properties of continuous and smooth curves while maintaining an organized topic structure. Subsequently a student, enabled with the goals and structure of a course in differential calculus, can refer to conventional texts to fill in and expand on subordinate details.**

***Index Terms*—branches of curves, visual derivative, curvature**

I. Introduction

DIFFERENTIAL calculus is primarily concerned with effects of small changes in related variables and the **direction** of curves. If two variables x and y are functionally related then the relationship can be imagined as a branch of a curve, say y = f(x). If the curve is continuous and smooth then it would be expected that small changes in the variable, x, would produce small changes in the variable, y.

A simple example of the functions being studied in calculus is a straight line of the form, y = mx + b. Here we are concerned with how small changes in the horizontal variable, x relate to the corresponding changes in the vertical variable, y. The change in x, $x_2 - x_1$, is called the **run** and is represented by the symbol Δx. The change in y, $y_2 - y_1$, is called the **rise** and is represented by the symbol Δy. The significance for a straight line of the ratio of the 'change in y' to the 'change in x' which is called the difference quotient can be observed in Equation 1.

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$$\frac{\Delta y}{\Delta x} = (y_2 - y_1) / (x_2 - x_1) = \frac{\text{rise}}{\text{run}} = m = \tan(\alpha) \quad \text{Equation 1}$$

where α is the angle of inclination of the line with the horizontal. Since the direction of a straight line is constant at every point, so too will be the angle of inclination, the slope, m, of the line and the difference quotient between any pair of points. In the case of a straight line vertical changes, Δy, are always the same multiple, m, of the corresponding horizontal changes, Δx, whether or not the changes are small.

However for curves which are not straight lines, the situation is not as simple. Select two pairs of points at random on a curve and the corresponding straight lines through these pairs will in general have neither the same direction, nor the same slope nor the same ratio of incremental differences. This can be observed for the circle in Figure 1

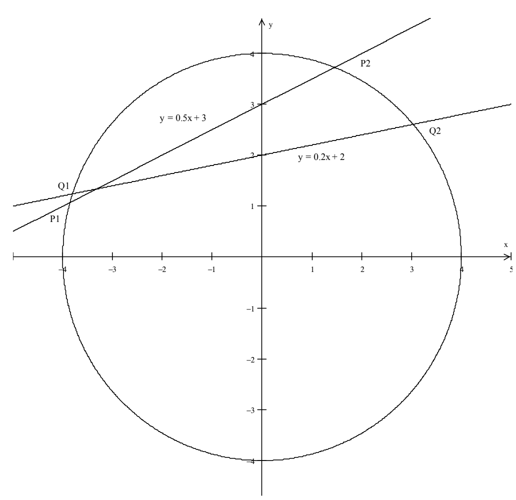

Figure 1 Pairs of points on a curve may not have the same slope

If the curve is smooth then at a fixed point, say P, the ratio of incremental differences for the tangent line will be close to the ratio of incremental differences on the curve near P. This fact, that is, that $\frac{\Delta y}{\Delta x}$ on the curve is approximately equal to the slope, m, of the tangent line is the basis of differential calculus.

The following graph, Figure 2, illustrates this principle. A circle of radius 5 is drawn with a line, T, tangent to the circle at the point P(3,4). The equation of the tangent line, T, is

$$(y - 4) = -\frac{3}{4}(x - 3) \quad \text{or} \quad y = -0.75x + 6.25.$$

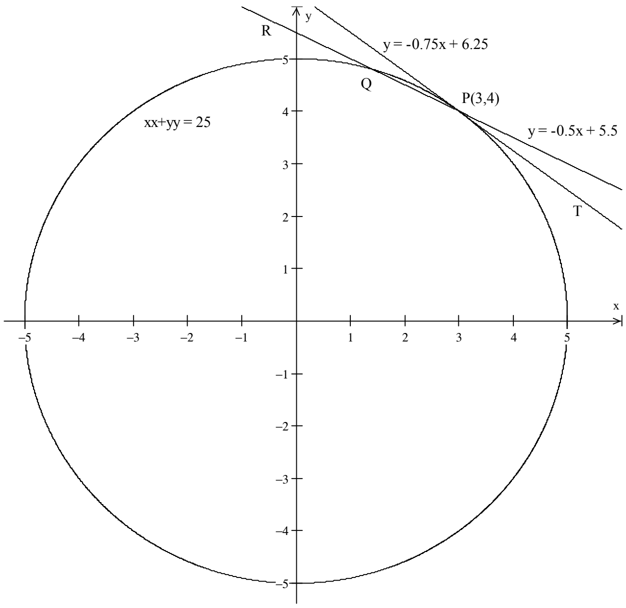

Figure 2 The rotation of a secant line until it coincides with the tangent

A secant line, R, is drawn through the point P which intersects the circle at the point Q. If the point of intersection, Q, is moved along the circle toward the point P, the secant line, R, will be forced to rotate clockwise. When Q coincides with P the secant line will become tangent to the circle and coincide with the tangent line, T, and acquire the same slope as T. This value, the slope of the tangent line at a given point, P, is a measure of the direction of the curve at P. This value is called the derivative of the describing function at the point P. An animation[3] of a rotating secant line approaching a tangent line can be found on the Internet.

It is seen that for a curve the direction of the tangent line changes from point to point, implying that the slope of the tangent line is a function of position on the curve. This function which is called the derivative function, m(x) is related at each point to, 1) the direction of the tangent line, 2) angle of inclination of the tangent line and 3) the ratio of incremental differences.

Another concern of differential calculus regards the **turning** of the curve. If at a point, the curve lies above its tangent line; that is, the curve turns upward, then the slopes of the nearby tangent lines increase as x increases and if the curve lies below its tangent line, then the slopes of the nearby tangent lines will decrease and the curve turns downward. Smoothly increasing slopes implies that the derivative of the slope is positive. If the original curve is described by the equation y = f(x) and the slope of the tangent line is described by the derivative function m(x) then we will need the derivative of the derivative to describe whether the curve is turning up or turning down. The derivative of the derivative is called the second derivative. The quartic curve in Figure 5 turns up, away from the tangent line then turns down and then turns up again.

On notation

When new concepts are discovered notation is needed in order to refer to these concepts. Differential calculus was crystallized by Isaac Newton in England and by G.W. Leibniz in Germany. Each devised his own notation. Currently Newton's notation has evolved to writing y(x) to represent the original function; $y'(x)$ to represent the derivative function which was previously called m(x) and $y''(x)$ to represent the second derivative. This notation is clean and easy to read in circumstances where there is only one independent variable.

On the other hand the notation used by Leibniz appears more cluttered but better describes the meaning and application of the derivative and enabled the concepts of calculus to develop and spread faster in Europe than in England. In the notation of Leibniz, the derivative was represented by the symbol, dy/dx and the second derivative by the symbol, d^2y/dx^2 . These symbols, lacking the name and parentheses do not convey the functional aspect of the derivatives. However the symbol dy/dx represents a number which magnifies or compresses small changes in x to produce closely the corresponding changes in y which can be written as dy = dy/dx dx. In the notation of Leibniz, the derivative appears as a quotient of the differential variables dx and dy.

This equation naturally extends to a formula, called the chain rule, for the derivatives of multiple compositions of functions. If y = f(u) and u = g(w) and w = h(x) then

$dy = \frac{dy\ du\ dw}{du\ dw\ dx}\ dx$. Stated in words, if we are considering functions of functions of functions, that is, chains of functions, the effect of making small changes in the initiating variable, dx, on the terminal variable, dy, can be easily computed by multiplying dx by the derivatives of each of the functions in the chain.

Some additional notes on notation will be needed. Individual points are described using an equal sign e.g. x = 6 or x = 7. Intervals whose endpoints are a and b are described using inequalities, e.g., $a < x < b$, which is most clearly read as 'x is between a and b'. Therefore, it will be seen below that the zeros, extreme points and points of inflection will be described with equalities. Additionally inequalities will be needed to describe the alternating intervals where the function is positive or negative, the function is rising or falling and where the function is turning up or down.

A definition of calculus

There is an assumption that holds in math courses that the author of the text has a clear idea of what constitutes calculus which certainly the student will also have by the end of the course. Is calculus the study of functions? Some very intelligent men have dreamed up examples of fantastic functions which most students do not need at the moment and probably will not need in the near future. Perhaps something could be gained by limiting the course content to clear examples which beginning students will need almost immediately. Nothing would be lost if the "terrors and horrors" about which the mathematician Hermite wrote were omitted or postponed. **Calculus** is the study of curves which are mostly continuous, mostly smooth and do not "wiggle" excessively. By "mostly continuous" I mean that the curve can have only a finite number of jumps or gaps in any finite interval. By "mostly smooth" I mean that the curve has a unique tangent line at every point except at only a finite number of cusps or corners in any finite interval. "Unexcessive wiggling" means that there can only be finite number extreme points in any finite interval. These are most of the curves that arise either from algebraic equations with one independent variable or the elementary transcendental functions. A student interested in science or engineering would lose nothing if he or she conceived of the functions being studied in the calculus class as such moderate curves.

On single-valued curves

Many of the curves that interest engineers are not single-valued. Since polynomials and rational functions are comprised of single valued operations, their corresponding curves are automatically single-valued. However, if a parabola is rotated even slightly on the Cartesian plane, it is no longer single-valued, and neither are general algebraic curves. If a polynomial is not monotonic, the curve of its inverse function is not single-valued. Naturally occurring curves such as the rose curves and hypocycloids lead us to realize that ultimately multivalued curves will have to be studied.

Mathematicians have realized that initially there is an advantage in restricting the study to single valued curves. For such curves, every x coordinate will have only one y-coordinate. Both of these coordinates will locate only one point (x, y). If the curve is smooth, at this point there will be only one tangent line and only one curvature or rate of turning. In the following we will focus on connected, single valued pieces of curves which are called **branches**. The circle is split by its horizontal diameter into a lower branch and an upper branch. Another example is the rose curve, shown in Figure 3, which is comprised of four branches. Each branch starts and terminates at a point of vertical tangency, P, Q, R or the origin.

The four branches are; one branch starting at the point P and proceeding counter-clockwise through the origin to the third quadrant and terminating at the point R, a second branch starting at R and continuing to the origin, a third branch starting at the origin and continuing counter-clockwise to Q, and the last starting at Q and proceeding still counter-clockwise through the origin returning to the initial point, P.

Defining the functions of calculus

Conventional calculus texts "define" functions as being single-valued and stress the concepts of domains and ranges. This is result of the successful, century long struggle by

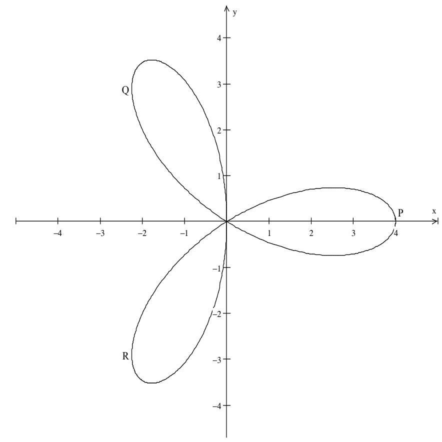

Figure 3 A rose curve

mathematicians to understand the dilemmas associated with infinite series forms. But these series forms are not in the immediate view of the calculus student and the all-inclusive definition confuses many students. Limiting the definition of a function to a branch of a smooth curve during this developmental period provides a student with a concrete concept while the computing dexterity and insight is acquired which will enable at the proper time, the consideration of the series forms. An overly abstract definition at this time does not provide the students with conceptual insight.

Let us examine the Dirichlet function which meets the conventional textbook definition of a function yet is so wild as to have no bearing in a course in beginning calculus. This function is defined as assigning the value 1 to all the rational numbers and having a value 0 elsewhere. This function is single-valued and has a domain and a range but is highly discontinuous and whose graph has no tangent lines or curvature. This function has no derivatives and is irrelevant and inappropriate in a beginning course in calculus but belongs appropriately in courses such as topology and real analysis. I am proposing that the definition provided to students in the calculus course focus on the continuous smooth curves that the student will need in their courses.

The concepts of domains and ranges are needed in courses such as topology and real analysis with rigorous logical requirements and where the domains and ranges can be complicated. The domains and ranges of the functions of calculus are usually simple sets excluding isolated points and intervals. All odd polynomials, can include very different curves, yet have identical domains and ranges. The focus should be on the shapes, the zeroes, the location of the peaks and valleys and the rises and falls and the turning.

I would like to use the word function to mean the curve or branch or branch of the curve but one does not differentiate a curve. One does differentiate the explicit form of the collection of operations that produce and represent the curve. In the interests of providing a meaningful and relevant definition for a major concept in the calculus course, the STEM (Science, Math, Engineering and Technology) community should agree to use the word **function in a calculus course** to mean an explicit algebraic form which describes a branch of a curve. This definition should be placed in big letters in the front of the book so there will be no way that a student could miss the importance of this major concept. The function will be used to compute the vertical position of points on the curves and the derivative of the function will enable the computation of the slopes of the tangent lines. Since in any discipline smooth, continuous relationships between two variables can be described by curves, a student who understands the study of curves should have little trouble applying his knowledge of curves and calculus to other quantitative disciplines.

What is a derivative?

The **point derivative** of the function describing a curve at a fixed point, P(x, y), on the curve is the value of the slope of the tangent line at a point. The derivative describes the local direction of the curve. This definition simply states what the derivative is; it does not state how to find its value in any particular case.

Many texts say that "the derivative is the limit of the difference quotient at P(x, y)." The "limit of the difference quotient" is a description of a process which leads to the value of the derivative. To understand this definition a student must have a clear idea of the meaning of the limit concept. Students who have only recently encountered the word limit may not see its significance in the definition of the derivative.

If a curve has a tangent line at a point, then this line has a slope which describes the direction of the curve. How the slope is computed is a separate issue. A limit is not needed to understand that at the peak of a smooth curve, the tangent line is horizontal.

The delta-epsilon constructions entailed in the definition of the limit concept confuses many beginning students who have not had time to clarify the concept. It is not pedagogically sound to define a new concept in terms of an unknown concept. What is needed is the understanding that the rate of increase of a function at a point is related to the slope of the tangent line.

The derivative of a function at a point as described above is a number. If a curve is described by a function, y = f(x) then the variation of the derivative value, that is, the slope, at different points on a curve is described by a new function, **the derivative function** denoted in Newton's notation by the symbol y′ = f′(x). Unfortunately many conventional texts create confusion by failing to distinguish the derivative at a point from the derivative function. Usually both are simply referred to as "the derivative." The number, f′(a) representing the numerical value of the derivative of the originating function y = f(x) at the point, x = a, is found by first computing the derivative function y' = f'(x) and then evaluating at x = a, this derivative function.

Graphical interpretations of the function values, the derivative values and the second derivative values

In the study of curves the original function, y = f(x) provides the vertical position of the point whose horizontal coordinate is x. The derivative function y' = f'(x) provides the direction of the tangent line of the point whose horizontal coordinate is x. The second derivative function y" = f''(x) provides a measure of the turning or concavity of the curve at the point whose horizontal coordinate is x. Unfortunately the geometric measure of turning, the curvature, κ, is not simply related to the second derivative.

Say we have a curve whose equation is y = f(x). The value of y:

- is positive when the curve lies above the x-axis,
- is negative when the curve lies below the x-axis and
- is zero at those special points, called zeroes, when the curve crosses the x-axis.

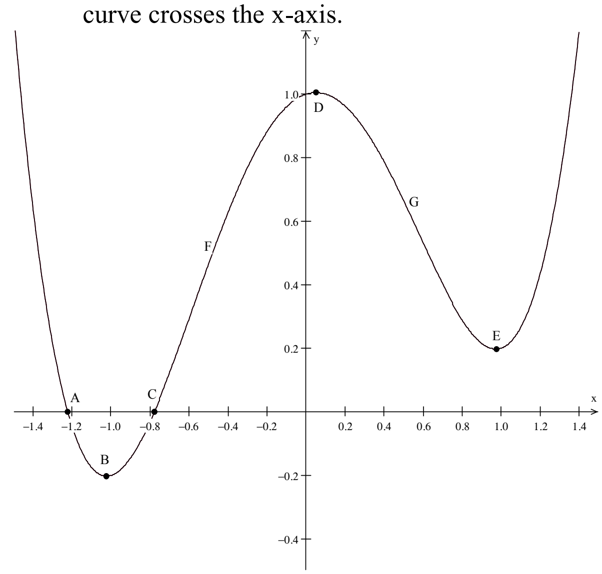

Figure 4 An asymmetric quartic

The curve shown in Figure 4 has zeros at the points A and C, has negative vertical values for the values of x between A and C and is positive for values of x either less than A or greater than C. In mathematical notation this is written;

f(x) = 0 at x = A and x = C,

f(x) < 0 when x is between A and C, and

f(x) > 0 when x lies either to the left of A or to the right of C.

The value of the derivative, y' = f '(x), describes the direction or slope of the curve at each of its points. If the values of y increase as x increases, then a smooth curve will be rising and the slope of a tangent line to the curve will be positive. If the curve alternately rises and falls, a situation I call wiggling or snaking, then the sign of the derivative will alternate. The value of the first derivative, y'(x), which is the slope of the tangent line to the curve:

is positive when the curve is rising,

is negative when the curve is falling and

is zero at those special points where its tangent line is horizontal.

The curve shown in Figure 4 has horizontal tangent lines at the points B, D and E. Note should be made that at the high points and valleys of all smooth curves, the tangent lines are horizontal. The curve rises for values of x between B and D and for values of x greater than E. The tangent lines slope upward for these same values of x between B and D and for values of x greater than E. The vertical values of the curve fall for values of x less than B and for values of x between the points D and E and the tangent lines slope downward in the same regions. In mathematical notation this is written;

$f'(x) > 0$ when either x lies between points B and D or to the right of point E,

$f'(x) < 0$ when either x lies to the left of point B or between points D and E, and

$f'(x) = 0$ at the points B, D and E.

The turning or concavity of the curve at each of its points describes how the curve lies with respect to its tangent line. A curve which does not cross its tangent line but lies above its tangent line is said to be turning up or to be concave up. Alternatively if the curve lies below its tangent line, the curve is described as turning down. If the curve wiggles or snakes, the curve can be seen as alternately turning up and turning down.

In an interval where the curve turns up, the slope of its tangent line is increasing. An increasing slope implies that the derivative of the slope which is the second derivative of the function must be positive. The value of the second derivative of y(x), which is symbolized in Newton's notation by y'':

is positive when the curve is turning up,

is negative when the curve is turning down and

is zero at those special points, called points of inflection, when the curve ceases to turn up and begins to turn down or vice versa.

Points of inflection can be characterized by:

The 2nd derivative, y'', equals zero,

The slope is either a maximum or minimum,

The tangent line crosses the curve,

The curve ceases to turn up and begins to turn down or vice versa.

As an example the graph of the curve $y = x^4 - 6x^2 + x + 7$ is shown below in Figure 5.

For this curve the derivative is: $y' = 4x^3 - 12x + 1$

and the 2nd derivative is: $y'' = 12x^2 - 12$.

The points of inflection for this curve occur when the 2nd derivative is zero which is either at point A which has coordinates (– 1, 1) or point B which has coordinates (+ 1, 3).·The curve is seen to turn up to the left of point A and to the right of point B. In the interval between points A and B the curve turns down. At the inflection point, A, the curve lies above the tangent line for x less than – 1 and below the tangent line when x is greater than – 1. At point B, the curve lies below the tangent line to the left of point B and above the tangent line when $x \geq 1$.

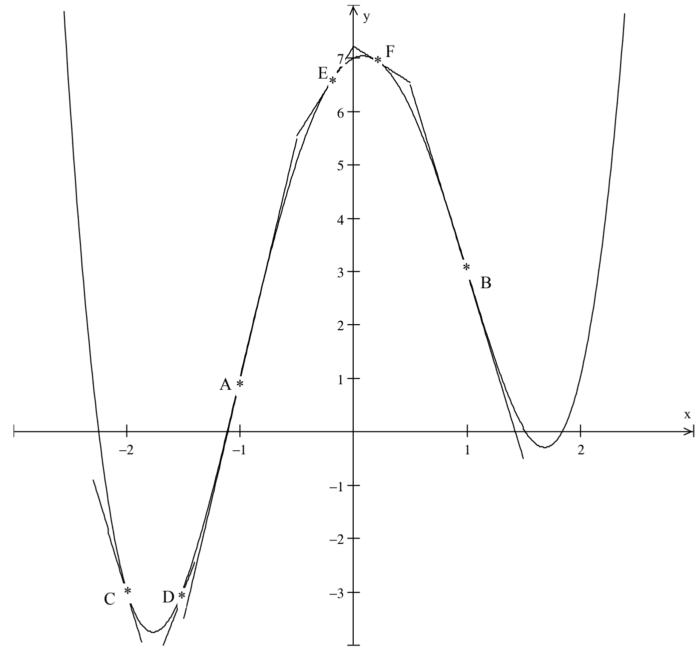

Figure 5 Tangent lines and concavity

It is always true that at a local maximum point of a smooth curve the tangent line is horizontal and the curve turns down. While at a local minimum point of the curve the tangent line is also horizontal but the curve turns upwards. This behavior is evident in Figure 5 in the intervals between points E and F and between points C and D.

As another example, the values of a sine curve $y = \sin(x)$ are positive in the first and second quadrants. Since in these two quadrants the curve is turning down, the values of the second derivative are negative. In the first quadrant, the sine is rising and the first derivative,

$y'(x) = \cos(x)$, is positive. However in the second quadrant the sine curve is falling and therefore the slopes and the first derivative are negative.

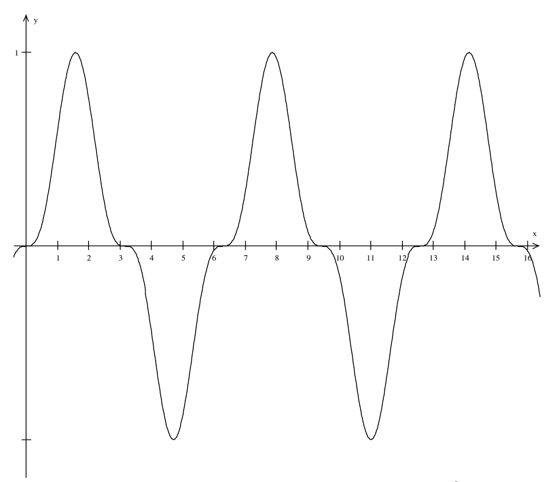

Figure 6. The curve of the function $\sin^3(x)$

It is possible that at a curve can have a point with a horizontal tangent which is neither a maximum or a minimum point.

Such a curve is the graph of the equation $y = \sin^3(x)$ whose zeros are all of the third degree as shown in Figure 6.

Evaluating the derivatives of polynomials and rational functions

To find the derivative of a polynomial or a rational function at a fixed point, P, imagine the process of seeking the limit of slopes formed by the secant lines through P and a nearby point Q as Q approaches P.

$$(y_Q - y_P) / (x_Q - x_P) = \frac{\Delta y}{\Delta x} \qquad \text{Equation 2}$$

Of course, when Q coincides with P, the differences will be zero and a rational function with a point gap of the form zero-over-zero results. The derivative is found by cancelling the corresponding factors in the numerator and denominator which will result in an expression which is not of the form zero-over-zero. Evaluating this resulting expression will provide the value of the slope of the tangent line at the point P.

Some properties of slopes and derivatives of curves

a) A horizontal line has a zero slope at every x.
b) Raising a curve vertically does not change the slope of the tangent line for a given value of x, that is, $\{y(x) + c\}' = y'(x)$.
c) The tangent lines at every point will rotate through an angle α when the curve is rotated by an angle α.
d) Multiplying a curve by a constant multiplies the slope at each x, by the same constant, that is, $\{c\, y(x)\}' = c\, y'(x)$.
e) Since a straight line has zero curvature, the horizontal location of points of inflection are unchanged if a straight line is added to a curve.
f) When a curve is rotated, usually the location on the curve of an extreme point will change. On the other hand points of inflection of a rotated curve will maintain their position on the curve.

Comparing Turning/Concavity to Curvature

Turning or concavity provides a crude description of the behavior of a curve at a point. Concavity only indicates whether a curve is turning up or down or whether the point is an inflection point. A more sensitive indicator would measure the rate of turning. One could start by examining circles. Since circles are completely symmetric, the rate of rotation of the tangent line is constant as the point of tangency varies on the circle.

If a point is moved some distance, Δs, on a circle of radius, R, the change in angle of the tangent line equals the angle subtended by the arc of the movement. The angle in radians, $\Delta\theta$, representing the change in direction of the tangent line is described by the equation,

$$\Delta\theta = \frac{\Delta s}{R} \quad \text{or} \quad \frac{\Delta\theta}{\Delta s} = \frac{1}{R}$$

This rate of change of direction, as measured by the angle of the tangent line, with distance along a circle is seen to be the reciprocal of the radius and is called the **curvature** of the circle. A larger circle possesses less curvature. A straight line maintains a constant direction and has a curvature equal to zero and does not turn. While circles have a constant curvature, the curvature, $\frac{\Delta\theta}{\Delta s}$, of other curves varies with position along the curve. Curvature is a true measure of the rate of change of direction of a curve.

As an example examine the parabola $y = x^2$ whose graph is shown in figure 7 below. The rate of turn of the tangent line as measured by $\frac{\Delta\theta}{\Delta s}$ is highest at the vertex. On the other hand the second derivative, $\frac{d^2y}{dx^2}$, which is commonly used to indicate concavity is constant. Of all the circles of curvature, the radius of the circle is the smallest at the vertex of a parabola; In this case the radius is 0.5. At the point (0.5, 0.25), the curvature is $1/\sqrt{2}$ and the radius of the circle of curvature is $\sqrt{2}$.

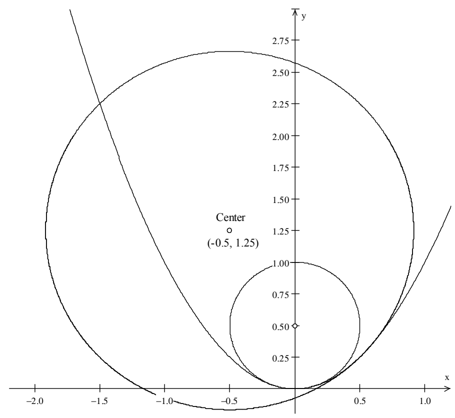

Figure 7 A parabola and circles of curvature

II. SUMMARY

Calculus is the study of mostly continuous and mostly smooth curves, which do not wiggle excessively and the equations, which describe branches of these curves. The strong correlation between algebraic equations and the properties of their curves is the focus of the study.

In particular, there is a concentration on the equations in explicit form $y = f(x)$ where f, which is called a function, represents a combination of single valued mathematical operations and whose graph is a branch of a possible multi-valued curve. At each horizontal coordinate of a point on these curves there will be a vertical position, a tangent line with a definite direction and a rate of turning of the tangent line. The position is determined by the function f(x), the direction is determined by the derivative function $f'(x)$, and the rate of turning of the tangent line is related to the second derivative of the function $f''(x)$. Only the values of the coordinates, $\{x, y(x)\}$ of a point on a curve and the direction $y'(x)$ of the tangent line are needed to determine the equation of the tangent line at any point.

At the high and low points of smooth curves the tangent lines are horizontal which means the first derivative must be zero. Therefore the maximums and minimums can be found by solving the equations of the condition that the first derivative must be zero. At the points of inflection the slopes of the tangent lines are extreme and therefore the second derivative must be zero. The points of inflection can be found by solving the equations of condition that the second derivative must be zero.

Of course it remains to the student to perform calculations by himself / herself to gain familiarity with examples of the many combinations that may occur and to acquire dexterity and familiarity in performing the calculations.

III. CONCLUSION

The main points have been covered. The focus is on properties of the branches of smooth curves. The definitions are in terms of the curves or their describing equations. The structure is governed by the properties of the curves, the kinds and forms of functions and the interactions of all the combinations of mathematical operations on functions. It will remain to another paper to treat the various rules of differentiation. A student should look forward to seeing how these ideas will extend to curves and surfaces in spaces of three dimensions.

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