# AC 2010-485: VISUALIZED PHOTOSTRESS IMAGES FOR STRESS CONCENTRATION INSTRUCTION

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# **Visualized Photostress Images for Stress Concentration Instruction**

### Abstract

The ever-increasing demand from industry for more sophisticated structural members and machine components requires a solid understanding of the concepts of different stresses and the behavior of members under loading. The optical method of reflected photoelasticity is utilized to achieve the goals concerning some of the learning outcomes of Strength of Materials and Design of Machine Elements courses. In particular, the paper addresses the use of photoelastic images to enhance the learning of stress concentration factors. Student reaction to the new approach is assessed and presented.

# Introduction

Mechanical engineering students are introduced to the concepts of stress and strain in a solid body through the Strength of Materials course. The principles and methods used to meet the learning objectives are drawn from prerequisite courses in statics, physics, and calculus together with the basic concepts of elasticity and properties of engineering materials. Perhaps, an analysis of the effects of stress concentrations is also discussed. In the first Machine Design course, junior mechanical engineering students learn to get a stress concentration factors (SCF), particularly with respect to fatigue, from a chart.

Several papers added an instructional prospective to enhance the teaching of mechanics courses to undergraduate students, the focus of this article. For example, the impact of demonstrations to acquaint students with the Statics concepts in the context of a real artifact was articulated in ref.<sup>1</sup>. A different approach in regard to teaching mechanics course came from Philpot et. al<sup>2</sup>. They presented examples of instructional media developed for the Mechanics of Materials Course utilizing computer in novel ways that offer the potential for improved instruction.

In the field of stress concentrations, the limited established theory does not give an insight for the understanding of the development of stresses in the vicinity of a discontinuity. Thus, experimental work is required to enhance the learning such as stress concentrations<sup>3</sup>. Strain gages are point measurements and the problems of discrete averaging are well documented<sup>4</sup>. In this article, photoelastic (photostress) images are proposed for the enhancement of learning stress concentration factor at the edge of a circular hole. The images are used to present the limitations of the theory that are not possible within the confines of textbooks.

# **Stress Concentration Factor**

In the intervals of three decades, a large amount of new material on stress concentration has become available, partly because of the growing capability of modern computer technology. The theory usually deals with infinite members. For example, Kirsch developed the theoretical stress distribution in the vicinity of a circular hole in an infinite elastic isotropic plate<sup>5</sup>. This theory predicts a stress-concentration factor (SCF) of 3.0 for

the hole with the maximum tensile and compressive stresses being 0 and 90 degrees from the horizontal axis of the hole, respectively. In 1930, Howland published the solution for the circular hole in a finite-width plate under uniaxial tension. In more recent years, solutions have been obtained for a wide variety of hole shapes under different loading conditions<sup>6</sup>.

In the case of repeated loading on any material, the presence of stress concentration is significant. Therefore, stress concentration, stress raisers, and stress concentration factors are investigated in a Machine Design course. Graphs or tables for SCF are presented in textbooks. Various fastening methods are also presented in machine design books. Due to the time constraints of a first Mechanics of Materials and Machine Design courses, there is generally insufficient time to verify the assumptions made in developing the theories with experimental verification. In addition, these courses normally deals with a combined loading topic in the analysis of stresses and strains produced by three fundamental types of loads: axial, torsional and flexural.

# Setting

The literature review showed that many articles have been written about the use of experiments that deal with the above combination of loads; the literature is bereft of describing the other combinations of loads that the students will encounter.

The topics of axial and bearing stresses are of fundamental importance in the first course on mechanics of materials and machine design. An instructor, with some assumptions, usually discusses it on a blackboard or computer only with freehand drawings or figures. It is probably not easy for him or her to give a second year engineering student a very clear picture about neither the assumptions nor the development of stresses and SCF in a structural member or machine component.

One of the objectives of application of more accurate SCFs is to achieve better-balanced designs of structures and machines. Therefore, conserving material, obtaining cost reduction, and achieving lighter and more efficient apparatus. Therefore, the proposed images are aimed to facilitate the understanding of these concepts for both learning and industry practice. The objectives of the proposed images are to demonstrate to the students the followings:

- The development of stress in the vicinity of a circular hole
- The effect of bolt preload on stress concentration around circular holes
- Interaction between axial, bearing, and contact stresses and their combined effects on reducing SCF
- An application of optics in engineering

Several images of a tension member with bolted connection plates are presented to illustrate the development of stress concentration as well as reinforcing the Saint-Venant's theory. The paper introduces the visualized photostress images in improving the understanding of SCF near notches/edges.

#### Physics

The optical method of reflected photoelasticity is utilized to achieve the goals about some of the above learning outcomes. Most engineering students learn the fundamentals of optics in a physics class. They are introduced to the geometrical and physical optics and most likely will not use the knowledge learned in the rest of the curriculum. A quick review of the fundamentals of polarized light is linked to the principal strains.

In a homogenous body, the index of refraction is constant regardless of the direction of light propagation or plane of vibration. However, in crystals the index depends on the orientation of vibration with respect to index axis. Certain plastics behave isotropically when unstressed but become optically anisotropic when stressed such as the photoelastic materials. The change in the index of refraction is a function of the resulting strain, analogous to the resistance change in a strain gage.

# **Optical Engineering**

In the second general physics course, engineering students learn the polarization of light by reflection and Brewster's law<sup>7</sup>. When a polarized beam propagates through a transparent plastic of thickness *t*, where X and Y are the directions of principal strains at the point under consideration, the light vector splits and two polarized beams are propagated in planes X and Y. If the strain intensity along X and Y is  $\varepsilon_x$  and  $\varepsilon_y$ , Brewster established that the relative change in index of refraction is proportional to the difference of principal strains or:

$$\left(n_{x}-n_{y}\right)=K\left(\varepsilon_{x}-\varepsilon_{y}\right) \tag{1}$$

Where *n* is the index of refraction. The constant *K* is called the strain-optical coefficient and it characterizes a physical property of the material. It is a dimensionless constant usually established by calibration and may be considered similar to the "gage factor" of resistance strain gages. The relative retardation<sup>8</sup>, in reflection, between the two beams in the X and Y directions is:

$$\delta = 2tK \left( \varepsilon_x - \varepsilon_y \right) \tag{2}$$

Combining equations 1 and 2, the basic equation relation for strain, hence the stress, measurement using photoelastic coating is:

$$\left(\varepsilon_{x}-\varepsilon_{y}\right)=\frac{\delta}{2tK}$$
(3)

In a circular polariscope, the relationship between the fringe order N and the relative retardation is:

$$\delta = N\lambda \tag{4}$$

Where  $\lambda$  is the wavelength. Thus, the difference between the principal strains can be calculated as<sup>9</sup>:

$$\varepsilon_x - \varepsilon_y = \frac{\delta}{2tK} = N \frac{\lambda}{2tK} = Nf$$
 (5)

The fringe value of the coating f contains all constants and its value is provided by the manufacturer of the photoelastic coating. Using Hooke's law, the difference between the principal stresses can be calculated from:

$$\sigma_{x} - \sigma_{y} = \frac{E}{1 + \nu} Nf$$
 (6)

Equation (6) represents the basic relationship underlying the photostress method of measuring as well as understanding the stresses in a member. It is clear that stress can be determined simply by observing the fringe order (color), since the mechanical properties of the specimen and the fringe value of the coating are known in a typical experiment.

In normal incidence measurements, the quantity determined is the difference of principal strains  $\varepsilon_x$ -  $\varepsilon_y$  which facilitates calculation of the difference of principal stresses  $\sigma_x$ -  $\sigma_y$ . In many practical applications (edges, corners), one of the principal stresses is zero. In these cases:

$$\sigma = \frac{E}{1+\nu} Nf \tag{7}$$

In the case of biaxial stress field that is beyond the scope of this paper, two measurements are needed to determine the individual principal stresses.

#### **Experimental setup**

In the photostress method, strain measurements are made by reflecting polarized light from the surface of a stressed part to which a photoelastic coating is being applied. A reflection polariscope is used to observe and measure the surface strains. The coating is a thin sheet of birefringent material, usually polymer. Starting with the unload test part, and applying the loads in increments, fringes will appear first in the vicinity of the hole. As the load is increased, new fringes appear near the joint. With further loading, additional fringes are generated in the highly stressed regions and move toward regions of less stress.

A typical commercial reflection polariscope consists of mounted polarizer and quarterwave plate attached to the common frame. They are mechanically connected so that they rotate in unison. The instrument may be hand-held or mounted on a tripod. The schematic for the optical system is shown in figure 1 [8, 9].

#### **Classroom lecture supplements**

The goal is to improve pedagogy and to visualize the development of stress concentrations rather than calculating them. However, it is essential to spend sometime in lecture to link physics and mechanics of materials topics, equations 1 through 7. This will enable the students to comprehend the development of stress in the member. The sequence of colored fringes produced by increasing stress is black (zero), gray, white, pale yellow, orange, dull red, purple, and blue (maximum). As a reminder to the students, this fringe identification sequence could be listed with every image. The purpose of each learning image is to acquaint students with the full-field stress regions. Thus, it is more advantageous and time efficient to have the images ready for the lecture than doing the experiments in the lab.

Any bolted joint connection, which can be used with a tensile machine, is sufficient for this task. The photostress images will be discussed combine contact, bearing, and axial stresses; these stresses were demonstrated by the use of joint connections as shown in Fig. 2. The effective length, width, and thickness of aluminum specimen used were 19", 5", and 1/8" respectively. The diameter of the hole was held constant at 5/16".

# Saint-Venant's region

The students learn about axial loading in their first engineering mechanics course, Statics. The photoelastic image of the specimen subjected to 1500 lb load is shown in Fig. 3. The Saint-Venant's region is apparent in the pale orange region. It shows the localized effects caused by the bolts. The dashed curve clarifies the assumption made in developing the  $\sigma$  = P/A equation. It separates the axial stress region from the assembly stress region.

At this stage, engineering students start appreciating the application of optics and light in solving practical problems. Emphasis is given to the interplay that exists between the understanding and practice of engineering mechanics in the learning process and beyond.

#### Nontraditional combined loading

This section deals with combined loading: axial, bolts load, and contact. Engineering students learn about joint connections in the Statics course. For example, the joint connections in a truss are usually formed by bolting the ends of the members to a gusset plate. Bearing stresses develop on surfaces of contact where the shanks of bolts are pressed against the sides of the hole through which they pass. Since bearing stresses are compressive normal stresses in nature, they can be uniquely added to the photoelastic image. The bolts and nuts should barely touch the stiffeners, (torque approximately .0), to minimize the effect of contact stress significantly. The oval in figure 4 approximates the black fringe area where the stress is zero in spite of the applied tensile load of 1500lb. Thus, the magnitude of compressive bearing stress equals the tensile axial stress. This will introduce the students to the combined axial and bearing stresses. It will also show the region where the hole can be drilled for optimizing (reducing) the stress in a member.

Figure 5 is the image for a load of 3500 lb; the area where axial and bearing stresses are equaled (black) has decreased significantly near the bolts. Thus, the rate of change of axial stress is grater than that of the axial one. The students can observe the marked region in which the stress level corresponds to the previous load of 1500 lb, but not in the center area.

Although contact stress is not normally covered in the machine design course, the author found that it was beneficial to introduce its effects optically. This can be achieved by introducing the bearing stress on the surfaces of contact between the head of the bolt and top thick plate and between the nut and the bottom thick plate. This bearing stress can be produced by increasing the torque on the bolt, which translates to a contact stress between the thick plates and the test thin plate.

The photoelastic images for the central hole, for different loads, are shown in figure 6. Initial observation of the pattern provides the students quick qualitative analysis of the overall stress distribution. In addition, as the number of fringes increases, the stress increases proportionally. The students appreciated the developments of SCF because these color bands (fringes) were closely spaced as the stress gradient near the hole became steeper. To illustrate the effect of bolts, the images of the same size hole that is one diameter from the bolted joint are shown in figure 7. The effect of bearing stress is more obvious at loads of a 1000 lb and 1300 lb due to the presence of some black fringes. The sequence of loads is kept the same in figures 6 and 7 for comparison reason. It is advantageous, but not necessary, to use double screens in the classroom to present the two figures. The reduction of SCF concept was further enhanced in figure 8. The images are for a same size hole that are both loaded at 1300 lb. The presence of some black fringe demonstrates the advantage of bolts load. In addition, the author used images from holes 1d (diameter), 2d and 5d from the center of the plate. However, they are not shown due to the limitation of the size of the paper.

#### Assessment

The author used the photoelastic images in teaching stress concentration factors. They were used in the design of machine elements course. The main objective was to introduce the effect of bolts on SCF. Therefore, the images were used after covering the fasteners and power screw chapter in the textbook. The sense was that the images helped the students to achieve the intended learning outcomes. The images generated a great deal of discussion and involvement by the students. This was accomplished by asking the students' questions about the stress regions as well as the level of reducing SCF. The students really enjoyed the supplementary images in learning reducing SCF.

One means of gauging the students' reaction to validity of this approach was to conduct a survey. Thirty-seven students were registered for the Machine Design course during three spring semesters. The standard deviation of the results between semesters is very small. Thus, the total response of the survey is listed in Table 1.

The results show the objectives of using photoelastic images were achieved. Further anecdotal evidence supporting the impact of using the images is the continuing discussions of these stresses as well as the experimental stress analysis method. Indeed, the discussion continued after class and the office, which indicate that it generated some extra interest in the subject. Below is some of the early typical student reaction:

"Visual aids always help to explain concepts and ideas shown in class."

"I learned the application of optics in my field."

"Pictures will always help me to remember the development of SC."

"Very helpful too see the images with and without the hole."

"Would like to compare different connections."

"Pictures gave me a better understanding of the bolt preload than the book."

"Helped me understand stress concentration and preload of the bolt."

"More pictures would be nice touch."

"Very helpful in seeing the changes in stresses and how they are affected by hole placement."

"Very helpful. Should use more in other chapters."

| The photoelastic images helped me understand:   | 5  | 4  | 3 | 2 | 1 | aver<br>age |
|---|----|----|---|---|---|-------------|
| The assumptions made in developing the axial stress formula, Saint-Venant's principle | 21 | 16 | 0 | 0 | 0 | 4.56        |
|   |    |    |   |   |   |             |
| The bearing stress effect in connections  | 18 | 12 | 6 | 1 | 0 | 4.27        |
| The combined loading concept (axial + bearing)  | 18 | 9  | 4 | 3 | 3 | 3.97        |
|   |    |    |   |   |   |             |
| The effect of assembly stress in practice   | 20 | 17 | 0 | 0 | 0 | 4.71        |
| An application of optics in engineering   | 14 | 12 | 9 | 0 | 2 | 3.97        |
| Stress concentration  | 24 | 12 | 0 | 1 | 0 | 4.62        |
| Effects of assembly stress on reducing stress concentration factors                   | 20 | 12 | 3 | 2 | 0 | 4.35        |

# Table 1. Results of student survey

Rating scale: 1 nothing 2 very little 3 modest amount 4 quite a bit 5 a lot

#### Conclusions

Examples of using photoelastic images for the Mechanics of Materials and/or Machine Design courses are presented. To understand the behavior of a member under loads, full-field visualization of stresses is highly recommended in teaching SCF. The photoelastic images can be used as a tool to supplement the approach normally used in lecture and textbook formats. The photoelastic fringes in the figures have characteristic behaviors that are very helpful in stress reduction interpretation. The fringe patterns are rich with information and insights for understanding the development and reduction of stress concentration factor.

The details of classroom implementation and assessment of the efficacy of this approach are presented. The assessment results indicate that the supplementary images helped the students to understand the development of basic axial, bolt preload, and stress concentrations. It also emphasizes the use of optics in the mechanical engineering field.

In parallel with the pedagogical benefits described, the overall presentation of photoelastic technique brings benefits as detailed in the ABET Criterion:

- An ability to apply knowledge of mathematics, science, and engineering
- A recognition of the need for and an ability to engage in lifelong learning

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Figure 1: Schematic experimental setup







Figure 3: Saint-Venant's region



Figure 4: Combined axial and bearing stresses



Fringe equivalent to the original 1500 lb load

Figure 5: Image for a load of 3500 lb



P = 1000 lb



P = 1600 lb



 $P = 2300 \, lb$ 



P = 3000 lb



P = 1300 lb



P = 2000 lb



P = 2600 lb









P = 1000 lb



P = 1600 lb



 $P = 2300 \, lb$ 



P = 3000 lb



P =1300 lb



P = 2000 lb



P = 2600 lb







Central hole (P=1300 lb)



Vicinity of bolts hole (P=1300 lb)

Figure 8: Comparison of SCF