Visualizing the Inherent Properties and Animated Responses of Vibrating Systems Based on Finite Element Modelling

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Abstract

Vibration is destructive in most mechanical systems and structures. It causes rapid growth of cracks, leading to fatigue failure of mechanical parts. Manufacturers set performance standards for their products to avoid excessive vibration. Vibration is an important topic in industry, and it is necessary for mechanical engineering students to learn how to analyze the responses of vibrating systems subject to various inputs.

Finite Element Analysis (FEA) has been adopted in the teaching of various mechanical engineering courses. This paper documents the development of a series of FE models as a supplement in teaching the engineering vibration course. FE models are developed in this paper for both single-degree-of-freedom (single-DOF) systems and multiple-degree-of-freedom (multiple-DOF) systems subject to initial displacement/velocity, harmonic excitation, impulse excitation, and arbitrary force excitation. The FEA features enable students to visualize the inherent properties of a vibratory system, such as the dependence of natural frequencies on the inertia, stiffness and dissipation elements, and mode shape associated with each natural frequency in a multiple-DOF system. FE animated responses of vibrating systems to different excitations help students understand the characteristics of various responses, such as transient responses and steady-state responses, resonance and damping effect on the responses excited by harmonic forces. As an application, an airplane has been modelled by using a three-DOF system (fuselage and two wings) in this paper for studying its inherent properties and vibration responses to various inputs.

1. Introduction

Vibrations are undesirable and harmful in most cases in mechanical systems and structures [1]. Noise, vibration and harshness (NVH) control, for example, has long been an important research in automotive industries. Vehicle NVH characteristics influence customer's perception of quality and comfort. The annoying oscillation as a car rides over a bumpy road conveys an impression of poor quality to the customers. Structure-borne cabin noise radiated from vibrating surfaces creates a nuisance to passengers. Knowledge about engineering vibration is desired for mechanical engineers to analyze, measure and control the harmful effects upon device performance.

The engineering vibration course is built on previous courses in mechanical engineering curriculum. The prerequisites include the dynamics and engineering mathematics (equations of motion and principles of energy in dynamics, Laplace transform and eigenvalue problem in engineering mathematics, for example). This introductory course covers the essential fundamentals of mechanical vibration analysis, including single-DOF and multiple-MDOF systems, free and forced responses, and vibration measurements and suppressions. Students are usually good at modeling vibration systems based upon dynamic analysis, setting up system differential equations, and finding corresponding solutions of the equations with specified initial conditions. However, it is not easy for many students to get insights of vibration responses obtained from solving the system equations.

Hand-on testing is recognized as an irreplaceable learning experience in engineering education. Various laboratory experiments have been employed in vibration courses to demonstrate related topics and phenomena. Ruhala [2] describes five forced-vibration experiments developed for an engineering vibration laboratory course. These experiments are built for measuring the transient or steady-state response of a lumped mass system with either single or multiple degrees of freedom. It is concluded that the laboratory experiments are effective in helping students understand the vibration theory and provide an increased level of intellectual excitement for the course. McDaniel and Archer [3] develop a full-scale experimental laboratory for teaching a mechanical vibration course. Forced vibration testing is employed to excite a one-story building constructed by students. The testing is designed to experimentally determine the building's natural frequencies, mode shapes, and damping. Excitations along the vertical and lateral directions are conducted in this experiment for demonstrating vibration mode shapes and improving computational model predictions, respectively.

Computer simulation has a variety of benefits when being applied in engineering education. One of the most attractive characteristics is that students can change design variables and observe corresponding changes in system responses. Computer simulation provides an alternative to some lab experiments which are traditionally performed by hands-on testing. FEA has wide applications in industries as a powerful tool for engineering modelling and simulation. In conjunction with experimental modal analysis, FEA has been widely employed to solve problems relating to engineering vibrations. FEA has also been integrated in the teaching of engineering vibration courses. Appropriate FE vibration models provide an efficient way to assist students in the learning of the inherent properties of vibration systems. Animations and graphical plots from FE modelling enable students to visualize the vibration responses and enhance their comprehension of vibration theories accordingly. Jenkins [4] develops a series of computer simulation modules for vibration course, including a topic on simulating an earthquake on a 4-DOF structure with actual acceleration data from 1940 E1 Centro earthquake. It is concluded that these modules can improve the students' comprehension of the interaction of multiple degrees of freedom. Baker [5] integrates FEA modelling in the teaching of a vibration class. Students develop a FEA program using MATLAB user function capability for calculating structure natural frequencies, mode shapes, forced harmonic responses, and other topics covered in the vibration analysis course.

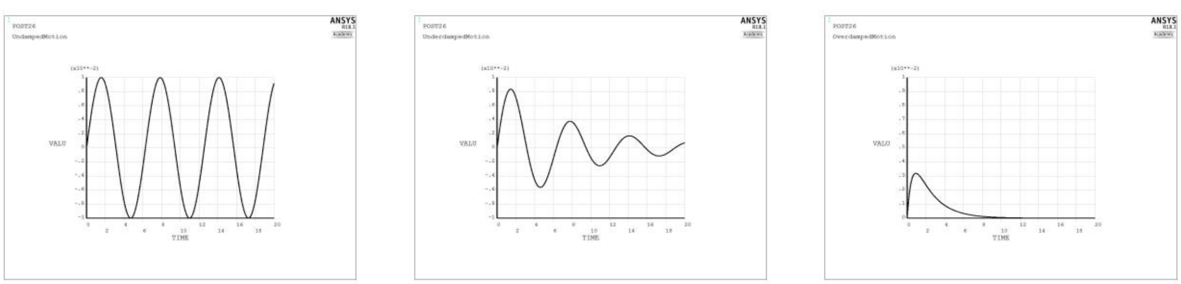
This paper documents an effort to make use of FE models as a means for students to comprehend inherent properties of a vibratory system and visualize responses to different excitations learned in the engineering vibration course. The developed FE models cover most topics in the course syllabus and demonstrate the characteristics of harmonic force excitation, arbitrary force excitation, damping effects on vibration responses, natural frequencies and mode shapes of a multiple- DOF system. As an application, an airplane has been modelled by using a three-DOF system (fuselage and two wings) in this paper for studying its inherent properties and vibration responses to various inputs. Student feedback to integrating FE visual results in vibration class is positive.

2. FE Models as a Supplement for Teaching Engineering Vibration

2.1 Free vibration of systems

Free vibration takes place under the action of the internal restoring forces and results from a nonzero initial displacement and/or velocity applied. A system in free vibration oscillates at its natural frequency, which is an inherent property of the system and is calculated by the stiffness, mass, and damping ratio.

Natural frequency and damping effect are two critical concepts in the study of free vibration responses. A spring-mass-damper vibrating system with a single DOF is built in ANSYS. Under the initial displacement and velocity applied, the free responses are plotted in Figure 1. Figure 1(a) shows the response with damping effect ignored. It is obvious that the harmonic vibration amplitude remains unchanged with time, which is one characteristic of undamped free vibration. Damping is introduced to the system for investigating the damping effect on vibration responses. Figure 1(b) shows the underdamped response in which the damping ratio is less than 1. The vibration amplitude, instead of remaining as a constant, decreases exponentially with time because of the factor $e^{-\zeta\omega_n t}$ in its analytical solution. Underdamped vibration is the most common case exhibited in many mechanical systems. Figure 1(c) shows the overdamped response with damping ratio greater than 1. It is clear that the overdamped motion does not oscillate, but rather return to its rest position exponentially. Even though being simple, this single-DOF FE model helps students enhance their understanding of the damping effects on the changes of vibration response.



(a) (b) (c)

Figure 1. Free responses of a system with a single DOF in which the mass m=100 kg, spring constant k=100 N/m, and variable damping coefficient c. (a) undamped free response, $\zeta = 0$, (b) underdamped free response, $\zeta = 0.125$, and (c) overdamped free response, $\zeta = 1.25$. The

scales used in the three plots are the same for comparison purpose.

The number of degrees of freedom of a vibrating system is determined by the number of moving parts and the number of directions in which each part can move. Most engineering systems are modelled as multiple-DOF systems. Airplanes, automobiles, and so on, all provide examples of vibrating systems well modelled by multiple-DOF analysis [1].

As an application of free response analysis, Figure 2 shows the vibration model of an airplane as a three-degree-of-freedom system with two side masses corresponding to the two wings and the middle mass for the fuselage. The stiffness connecting the middle mass to either side mass corresponds to that of the wing and is calculated as a function of material Young's modulus, cross section properties, and the wing length. FE vibration analysis has been conducted for studying responses of the system subject to various excitations.

Without losing generality, transient free responses of the system subject to an initial displacement $x(0) = [0.2\ 0\ 0]^T$ are plotted as functions of time in Figure 3.

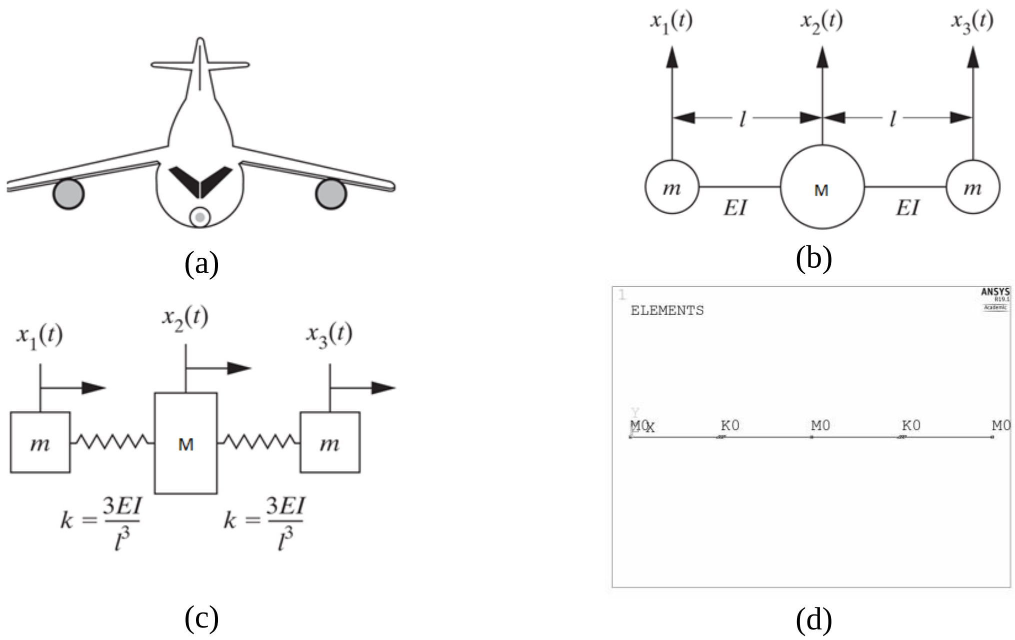
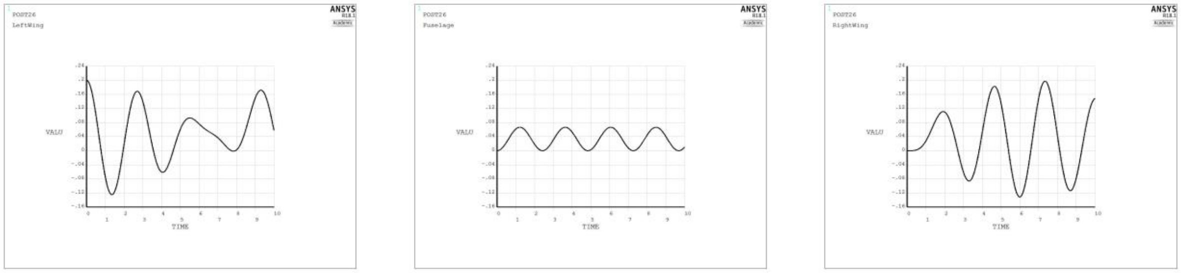


Figure 2. Vibration model of an airplane. (a) vertical wing vibration; (b) lumped mass/beam deflection model; (c) spring–mass model. m=3000 kg, M=12000 kg, and k= 13455 N/m. (d) FEA model. The masses and springs are simulated by MASS 21 and COMBIN 14 element in ANSYS, respectively.



(a) (b) (c)

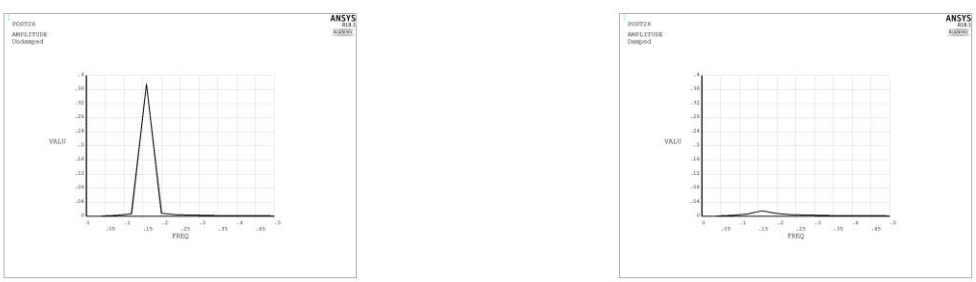
Figure 3. Free responses of an airplane with three DOFs. (a) left wing, (b) fuselage, and (c) right wing. The scales used in the three plots are the same for comparison purpose.

2.2 Response to Harmonic Excitation

Resonance is a critical concept related to forced vibration and it occurs when a periodic external force is applied to a system having a natural frequency equal to the driving frequency. Resonance will cause large displacements which may exceed the elastic limits and cause the structure to fail. Damping is of great importance in limiting the amplitude of oscillation at resonance. Students learn to understand resonance phenomena and its effects by applying harmonic excitation to the above-mentioned single DOF model.

The frequency of the applied harmonic excitation ranges from 0.05 to 0.5 Hz. Figure 4 shows the changes in the response magnitude over this frequency range for both undamped and damped systems. The peak response happens at the natural frequency of the system which is about 0.159Hz in Figure 4, agreeing well with the analytical results from hand calculations.

It is important from design point of view to note how the amplitude of the forced response is affected by the damping ratio. Figure 4 illustrates clearly the effect of damping on the response magnitude: the response magnitude near to the natural frequency of the system is very large without damping, but reduces significantly with increasing damping ratio.



(a) (b)

Figure 4. Harmonic forced responses of a system with a single DOF in which the mass m=100 kg, spring constant k=100 N/m, and variable damping ratio ζ. (a) undamped system, $\zeta = 0$, and (b) damped system, $\zeta = 0.125$. The scales used in the plots are the same for comparison purpose.

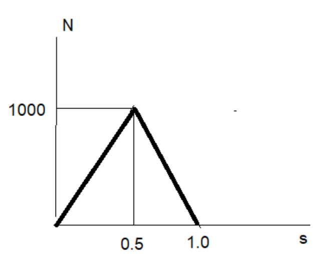
2.3. Impulsive and Arbitrary Forced Response

The response of a single DOF system to an arbitrary, general excitation of varying magnitude can be derived from the concept of impulsive response. An impulsive excitation is a force that is applied for a very short length time. For example, the force acting on a car when running over a speed bump. The impulse of a force is the integral of the force over the time interval for which it acts and provides a measure of the strength of an applied impulsive force. The response of a system to an impulse is identical to the transient free response of the system to certain initial conditions in nature.

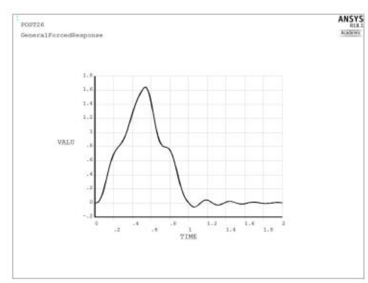
It is not hard for students to understand the response to an arbitrary force excitation (step force, ramp force, for example) by dividing the exciting force up into infinitesimal impulses, calculating each impulsive response and adding the individual response for the total response. Convolution integral is the most important mathematical concept in this application. The convolution of two piece-wise functions, $f(t)*g(t)$, has been introduced in a previous engineering mathematical class. Students could solve the convolution integral, or Duhamel integral, for some specified inputs by using Laplace transform and inverse Laplace transform.

FE modelling, on the other hand, provides students a visual insight into the mathematical results. To simulate the arbitrary forced response, a triangle-shaped force as shown in Figure 5(a) is applied to a single DOF vibrating system. The transient response is shown in Figure 5(b). By studying the transient response, students enhance their understanding of some quantities used as a measure of the quality of the response. One of such quantities is the overshoot, defined as the largest value of the response exceeding the corresponding steady-state value. Another quantity is the setting time, defined as the time it takes for the response to get and stay within a certain percentage of steady-state response. Furthermore, students are instructed to change damping coefficient in the FE model and observe the changes in the response quality.

As an application, impulsive response analysis is applied to the above airplane model. Suppose that the airplane hits a gust of wind which applies an impulse of $3\delta(t)$ at the end of left wing. The transient responses of the fuselage and two wings are plotted in the Figure 6. Students are encouraged to apply other types of arbitrary excitations to the airplane model, including step function, square-pulse function, etc.

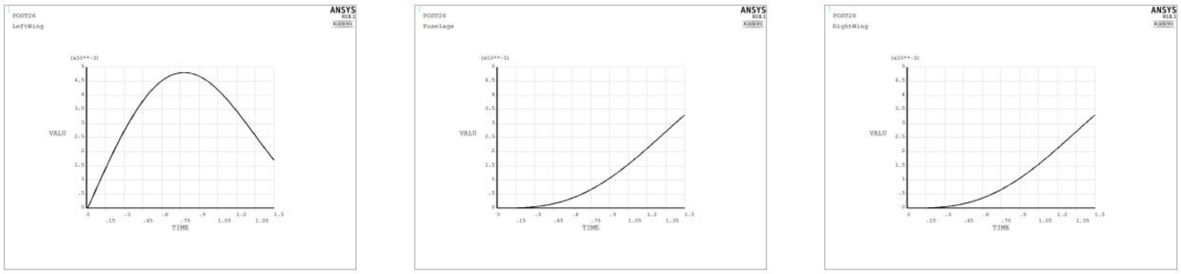


(a)



(b)

Figure 5. Arbitrary forced responses of a system with a single DOF in which the mass m=100 kg, spring constant k=100 N/m, and damping ratio $\zeta = 0.125$. (a) Arbitrary exciting force, and (b) forced response of the system.



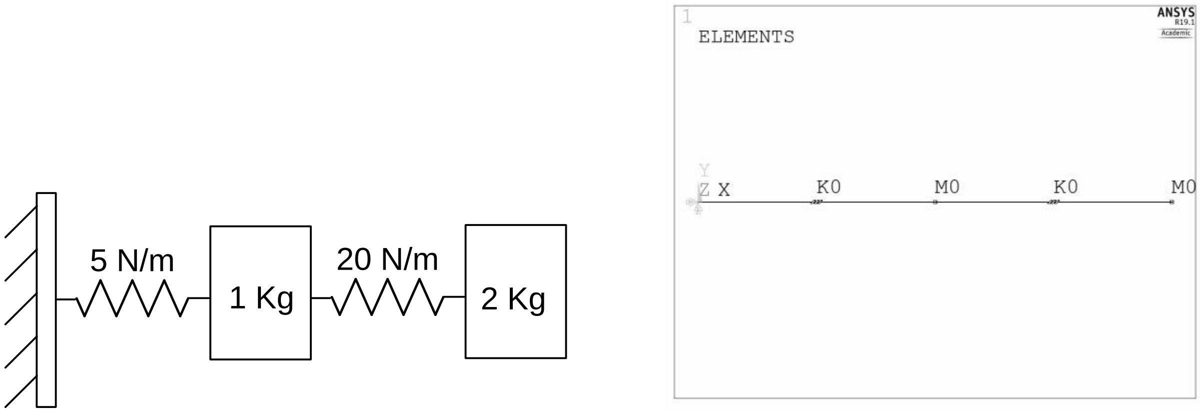
(a) (b) (c)

Figure 6. Impulsive responses of an airplane and wing system subject to 3δ(t) at the end of left wing. (a) left wing, (b) fuselage, and (c) right wing. The scales used in the three plots are the same for comparison purpose.

2.4. Modal Analysis

In moving from single degree of freedom systems to two and more degrees of freedom systems, two important physical phenomena result. Multiple-DOF systems have more than one natural frequency, and for each of the natural frequencies, there corresponds a natural state of vibration with a displacement configuration known as the mode shape. A mode shape is a vector that describes the relative motion among multiple degrees of freedom. Coupling due to spring forces is also an important concept in the multiple-DOF analysis.

Most students know how to apply Newton's Law or energy principles to formulate the equations of motion of multiple-DOF systems, how to identify the mass, damping and stiffness matrices from the equations of motion, and how to apply the mathematical concepts of eigenvalues and eigenvectors of computational matrix theory for vibration responses, However, it is hard for students to grasp the engineering insight of interactions among multiple degrees of freedom. Figure 7(a) depicts an undamped system with two degrees of freedom and the corresponding FE model is shown in Figure 7(b). Students also conduct analytical modal analysis based on hand calculations, including decoupling the two degrees of freedom through forward transformation, solving the resulting equations in the modal coordinate system, and transforming the results back to the physical coordinator system. By comparison, FEA results agree well with those from analytical analysis. See Table 1. Furthermore, the computed mode shapes are animated in ANSYS post-processing in class to assist students to visualize the relative motions between the two masses.



(a) (b)

Figure 7 (a) Two degrees of freedom system, (b) FEA model.

Table 1 Comparison of analytical results and FEA results

	FEA Results	Analytical Results
Natural frequencies (Hz)	0.1944 and 0.9213	0.1943 and 0.9213
Mode shapes	$[1\ 1.175]^T$ and $[1\ -0.425]^T$	$[1\ 1.176]^T$ and $[1\ -0.426]^T$

As an application, similar FE modal analysis is applied to the airplane model. It is found the three natural frequencies are 0.038 Hz, 0.337 Hz and 0.413 Hz, respectively. Again the three modal shapes are animated in the post-processing. Students are encouraged to take a further step to perform modal analysis for other multiple DOF systems (horizontal vibration of a four-story building, for example).

3. Student Feedback

It is imperative to conduct a survey for assessing the efforts of introducing the FE models in the engineering vibration course. Four questions are designed in the survey to measure the benefits of incorporating FEA in class teaching. Three questions are as follows and the last question asks for suggestions about these FE models developed for this class.

(1). FEA models help me understand the topics discussed in the vibration class.
(2). FEA models improve my critical thinking in vibration analysis.
(3). My techniques and skills of FEA modelling have been improved.

Figure 8 shows the survey results. Many students suggest to have more realistic engineering problems involved in the FE modelling in their answers to the fourth question.

The first question is the primary intention of integrating FE models in the class teaching. Responses to this question show that students benefit from these FE vibration models, which provide a visually oriented insight into engineering vibrations. The third question measures the contribution of this innovative teaching effort to one of the student outcomes in ABET Criteria which emphasizes on the ability to use the techniques, skills, and modern engineering tools necessary for engineering practice. Responses to the second and fourth question indicate that more realistic engineering problems should be integrated in FE modelling for fostering critical thinking to promote professional development.

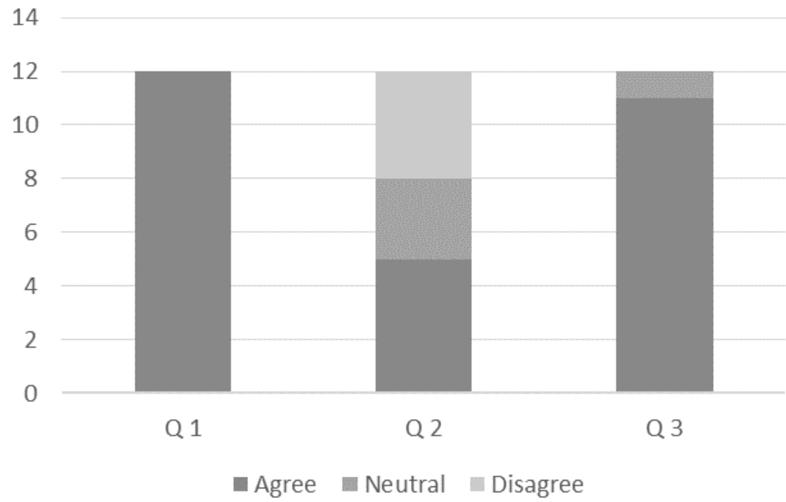


Figure 4 Student assessments of the FEA modelling on their learning

4. Conclusions

This paper documents the development of a series of FE models as a supplement in teaching the engineering vibration course. FE models developed in this paper include single-DOF and multiple-DOF systems subject to initial displacement/velocity, harmonic excitation, impulse excitation, and arbitrary force excitation. As an application study, an airplane has been modelled by using a three-DOF system in this paper. Students benefit from FE modelling in grasping physical insights of engineering vibration. Student feedback on integrating FE modelling is positive.

Acknowledgements

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Biographical Notes

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