

AC 2007-1362: WE CAN DO BETTER: A PROVEN, INTUITIVE, EFFICIENT AND PRACTICAL DESIGN-ORIENTED CIRCUIT ANALYSIS PARADIGM IS AVAILABLE, SO WHY AREN'T WE USING IT TO TEACH OUR STUDENTS?

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Abstract

Circuits and electronics textbooks today are arguably more attractive than past editions and are bolstered by a plethora of supplements and software; yet, when it comes to the core circuit analysis methods that every student must learn, how much real progress has been made? Learning and being able to apply circuit analysis fundamentals well is foundational, but too often engineering graduates find that the analysis techniques they learned in school seem to lack practical application on the job. This is especially true when it comes to the understanding and analysis of analog circuits. As result, young engineers find themselves uncomfortable in tackling needed analog designs. Even experienced engineers realize that there are probably more efficient ways of solving design problems, but often feel they lack the time to pursue them. All the while, globalization has made us keenly aware of the need for efficiency in electronic design. Despite the dominance of digital electronics, it can be argued that analog expertise and insight are even more critical for designing such things as high frequency communications or advanced power systems. In this paper the author calls for a paradigm shift in how analysis is taught by adopting an already proven unified body of techniques into our textbooks and lectures that heretofore have been largely unutilized. These techniques, pioneered by emeritus Caltech Professor R. David Middlebrook and others, and dubbed D-OA (Design-Oriented Analysis), needs to be mainstreamed into our educational consciousness and taught to our students. In a real sense they are intellectually liberating! The practical value and student acceptance of these methods have already been established in industry and in some universities, such as the author's institution. Examples based on common circuits used in textbooks are used to illustrate the advantages these techniques have over traditional methods

Introduction

One day I asked my friend Jon Parle, Principal Engineer at the John Fluke, Inc. and design guru of a number of Fluke's successful hand held meters, what was one of the most important skills he looked for in an engineering graduate. He quickly responded that what he looked for was someone who had a good intuitive grasp of how electronic components and circuits worked. He said that when it came to being a good designer, intuition was key to being able to use available theories and tools to efficiently create successful designs. Students in engineering programs usually could learn the theories of circuit analysis and the powerful CAE tools like MATLAB^{®1} and MultiSim^{®2}, but when it came to apply them effectively to real world

problems, they often struggled. One reason for this, he thought, was that so much focus went into mastering the mathematics of analysis and how to use software packages that emphasis on intuitive understanding lagged. I easily resonated with his remarks, having had a similar experience in my own formal engineering education and having worked in industry for over a decade in electronics where I quickly learned the truth of his words. This reinforced my resolve to seek all possible means of instilling as much intuitive grasp in my students as possible even while they learned the necessary mathematically based analysis techniques.

I had been teaching undergraduate electronics for a few years when I heard a compelling paper at the 1991 FIE Conference. The speaker was R. David Middlebrook, Caltech Professor and internationally known authority on power electronics. What impressed me about his presentation was the demonstrable practicality of his ideas for electronic circuit design. Essentially he was presenting a new way of thinking about design analysis, a new paradigm if you will, that drew the focus away from the mathematical complexity that can sometimes arise in real world problems and replaced it with a way of doing analysis that supported an intuitive perspective, the very thing my friend Jon had emphasized. He referred to his approach as Design-Oriented Analysis (D-OA). His premise was that the only kind of analysis worth doing is that which is useful for design, i.e., the kind that would provide the maximum amount of useful information possible for the smallest amount of work. He made the point that a designer should seek to minimize mathematical complexity while maximizing useful design-oriented information.

One could say that many past approaches to circuit analysis have tended to focus on the math – we could call it M-OA (Math-Oriented Analysis). They have tended to make use of matrix solutions and unstructured formulations of circuit equations that quickly can lead to algebraic paralysis. One possible reason for this is that some of the early leaders in the development of circuit analysis methods were mathematicians, like the famous Hendrik Wade Bode, whose seminal work *Network Analysis and Feedback Amplifier Design*³ was published in 1945.

Middlebrook's ideas are attractive, but are they practical? What specific, concrete methods can be used to actually implement this D-OA paradigm? Fortunately, the groundwork has already been laid. Over many years, Dr. Middlebrook developed a suite of practical techniques, concepts, and theorems that have given legs to his ideas.^{4,5,6,7,8,9,10} For years he has taught hundreds, if not thousands, of working engineers in a course entitled "Structured Analog Design" where his techniques are explained and illustrated. More recently, he has produced a Data DVD ROM containing his entire three day course (19 hours).¹¹ Examples of chapter titles are 'Using Normal and Inverted Poles and Zeros', 'Improved Formulas for Quadratic Roots', 'The Input/Output Impedance Theorem', 'The Extra Element Theorem', and 'The Feedback Theorem'. I have been using his D-OA approach in my classes with success, and have been able to customize his ideas for my circuits and electronics classes.

One of the core techniques embodied in D-OA is how to formulate circuit equations in such a way that their very structure provides maximal physical information about the circuit being analyzed. Unlike in the conventional approach, in which circuit equations are solved simultaneously, in the D-OA approach the equations are solved sequentially by Thevenin/Norton

and other circuit reduction techniques, a process called “doing the algebra on the circuit diagram.” These intentionally structured equations are called “Low-Entropy Expressions”. Their name is borrowed from physics. Entropy can be conceptualized as a measure of disorder in a system. When the entropy is high, order is low. When entropy is low, order is high. For example, the atoms in a large crystal grown in lab are well-ordered and as a system has low entropy, but the same atoms in a gaseous state at high temperature exhibit random orientation, exhibiting high entropy. In Low-Entropy circuit analysis expressions, the various components are arranged in such a way as to provide intuitive insight into how they contribute to the quantity being expressed, whether it is a voltage gain or an impedance. In this way, they are more useful for design.

Dr. Middlebrook is making available his D-OA paradigm on his website <http://www.RDMiddlebrook.com>. This summary of his techniques may be freely quoted, downloaded, or reproduced.

What I seek to do in the remainder of this paper is to illustrate the D-OA paradigm by taking examples of circuit analysis from current circuits and electronics textbooks and showing how by using some of Middlebrook’s techniques, they can be recast into more useful and intuitively meaningful Low-Entropy expressions. An exhaustive review of this topic is not possible here, but hopefully by the end of this paper, you will be motivated to begin your own process of incorporating these powerful methods into your teaching repertoire.

A Low-Pass Circuit

Figure 1 illustrates an exercise taken from an electronics textbook chapter dealing with frequency response¹².

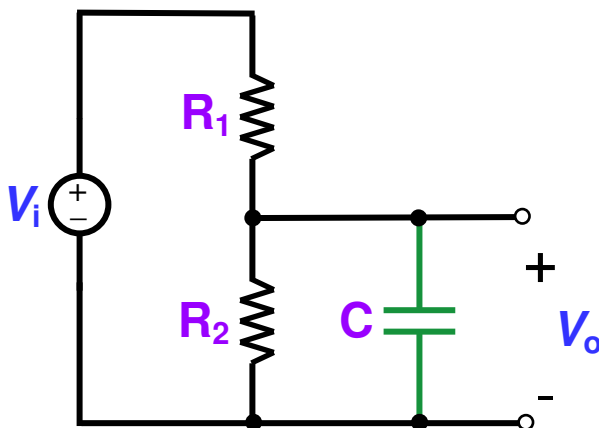


Figure 1 Find the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$ for the STC network shown

The textbook states “..using usual circuit-analysis techniques, one derives the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$.” The answer for this exercise is given as:
$$T(s) = \frac{1/CR_1}{s + 1/C(R_1 \parallel R_2)}$$

The “usual” circuit-analysis technique used is the application of a voltage divider expression utilizing the impedance expressions for R and C of the parallel impedance of R_2 and C ($Z_{R_2||C}$) divided by the sum of that impedance and R_1 , i.e.,

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_{R_2||C}}{Z_{R_2||C} + R_1} = \frac{\frac{R_2}{R_2Cs + 1}}{\frac{R_2}{R_2Cs + 1} + R_1}$$

After some more algebra the answer given above emerges. A question that should be asked is “what is the physical interpretation of this transfer function?” For example what kind of circuit is it? You may be able to recognize this as a low pass circuit since there is a ($s + \text{Constant}$) term in the denominator. You may also be able to identify an expression for the corner frequency as $1/[C(R_1||R_2)]$. However, can you readily identify the low frequency gain? That is not nearly as obvious. Further algebra is needed to find it. Though simple, this represents a high entropy formulation. A better approach would be to first replace the circuit comprised of the voltage source with the two resistors with its Thevenin equivalent so that then an explicit simple RC circuit is seen as shown in **Figure 2**.

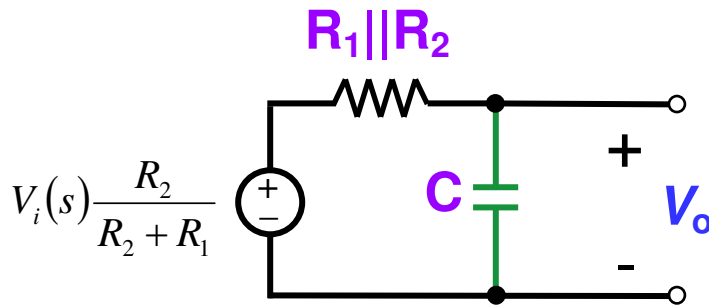


Figure 2

Applying voltage division to this new form of the circuit yields:

$$\frac{V_o(s)}{V_i(s)} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{\frac{1}{Cs}}{\frac{1}{Cs} + (R_1||R_2)} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{1}{1 + C(R_1||R_2)s} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{1}{(1 + s/\omega_o)}$$

where $\omega_o = 1/C(R_1||R_2)$

The physical interpretation of this form of the transfer function is more easily recognized. First, when the frequency is zero, the second term is unity so we can know that the zero frequency magnitude is the first term which is recognized as a voltage divider. The corner frequency ω_o is explicitly shown as the constant that is the denominator of the s term. The frequency dependent term is a readily recognizable form of a single pole. It is very important to note that the frequency-independent and frequency-dependent portions are explicit and separate. This, then, is a **Low-Entropy Expression**. It is clear how the two resistors and capacitor affect

the frequency behavior of the circuit, which exemplifies a **Design-Oriented Analysis**. In fact, in general, we can henceforth define the general low entropy form of a low pass transfer function as

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{(1 + s/\omega_o)}$$

where K is the zero frequency gain and ω_o is the 3dB corner frequency, which is always $1/\tau$ where τ is the circuit time constant. So the message is if you can look at a circuit and recognize it as a low pass circuit, then you can immediately write down the form of the transfer function and then find the two necessary constants K and τ . Recognition of this example circuit as low pass is straightforward by mentally analyzing what the output voltage will be when the frequency is zero and when it is infinite. In this case it is clear that for zero frequency the output is finite and when the frequency is infinite the output is zero. Since there is a single reactive component we can then conclude that this is a Single Time Constant (STC) low pass response. Equipped with this knowledge, it is no longer necessary to do any algebra at all – just write down the form and find the two needed constants.

Low-Entropy Summary of First-Order Responses

Now let's build on the last section to create a concise way to capture the essential transfer function characteristics for first order responses in low entropy form. We have already looked at the low pass RC circuit, whose general form is shown in **Figure 3** alongside its high pass counterpart. What *about* the high pass RC version? One electronics textbook shows its transfer function as¹³

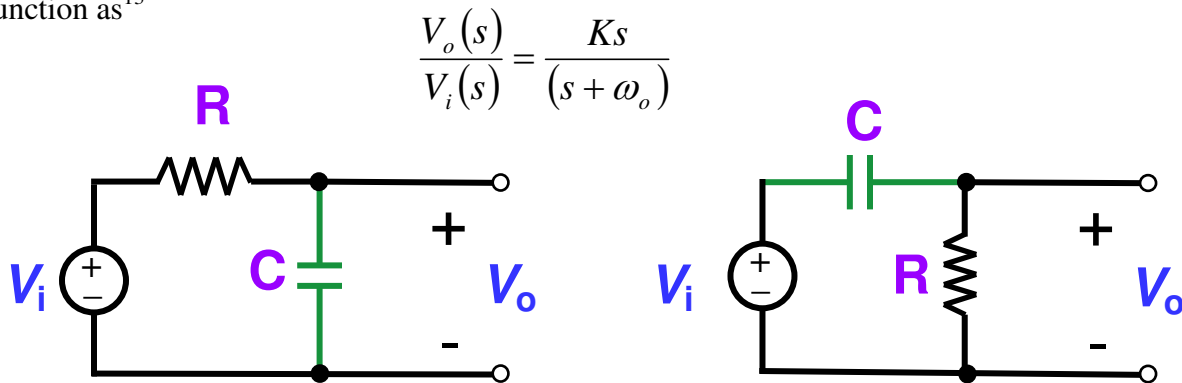


Figure 3

This form shows a pole at ω_o and a zero at 0. This is *not* a low entropy form. By simply dividing numerator and denominator by s we get the alternative form

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{(1 + \omega_o / s)}$$

which is a low entropy expression. Why? Because when the frequency is zero ($s = 0$) the magnitude is zero and when the frequency is infinite, the magnitude is K , meaning that K represents the high frequency magnitude. This is recognized as a high pass response. Just like the case of the low pass version, once we intuitively recognize this circuit as high pass by mentally testing the circuit for the two frequency extremes, we can write down the form of the transfer function and then simply find the two necessary constants, the high frequency gain and the time constant ($\omega_o = 1/\tau$).

Let us now look carefully at the Bode plots for these two basic first order circuits. **Figure 4** shows both the magnitude and phase responses of these circuits as rendered in MATLAB[®].

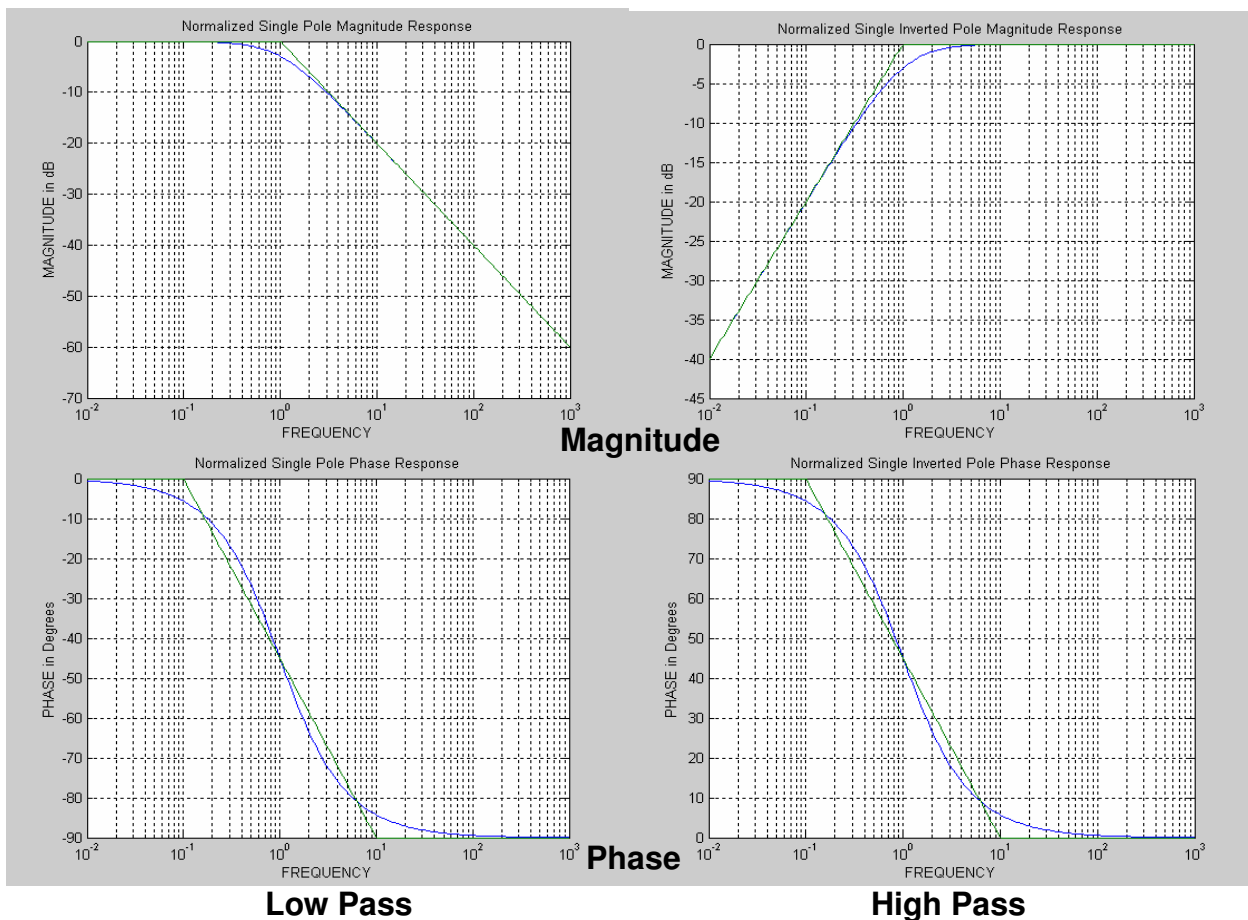


Figure 4 – low and high pass Bode plots

Note that the low and high pass magnitude responses are the same *except* for the fact that the low pass response (the pole) is concave downward with *increasing* frequency whereas the high pass response is concave downward with *decreasing* frequency. This is why it is logical to call the high pass response an **inverted pole**. With this label we recognize that it looks like a normal pole with reversed orientation on the frequency axis. Note also that the phase responses are also the same in that they pass through a -90° phase shift with increasing frequency, with the difference being that the **inverted pole** starts at $+90^\circ$ and transitions down to 0° in contrast to the normal pole starting at 0° and transitioning to -90° .

The illustration of the low and high pass response can be extended to two more first order responses, the zero which is represented by the general form of

$$\frac{V_o(s)}{V_i(s)} = K\left(1 + s / \omega_o\right) \quad \text{for a normal zero and} \quad \frac{V_o(s)}{V_i(s)} = K\left(1 + \omega_o / s\right)$$

for the **inverted zero**. As in the case of the low pass and high pass forms, the zero and the inverted zero responses, when recognized for a given circuit, can be determined by writing down the general form and then finding the same two necessary constants of the infinite or zero frequency magnitude and the time constant, which is tantamount to finding the corner frequency. The correlation of the magnitude and phase responses is similar to those of the high and low pass responses. Students should be shown these explicit relationships and taught how to use intuition to recognize these types of circuit responses. They should also be taught their asymptotic approximations and how they relate to the actual responses and to each other. **Figure 5** shows the complete set of first order Bode plots, utilizing the asymptotic forms.

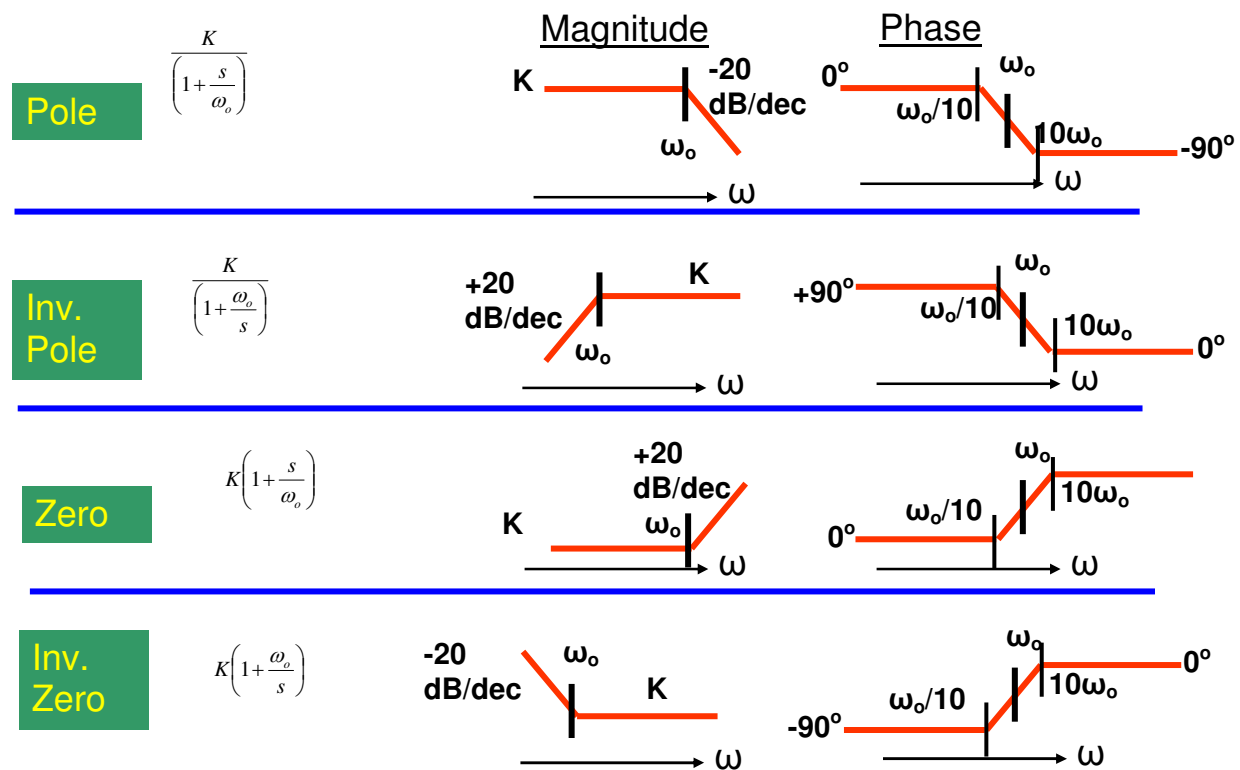


Figure 5 – first order asymptotic Bode forms

Improved Formulas for Quadratic Roots

Any frequency response transfer function can be constructed with a leading constant and a combination of first and second order frequency dependent factors. There are several ways to represent a second order factor. One way is written as $(j\omega)^2 + j2\zeta\omega_o\omega + \omega_o^2 = s^2 + 2\zeta\omega_o s + \omega_o^2$.¹⁴ Another form is written as

$$1 + \frac{1}{Q} \left(\frac{s}{\omega_o} \right) + \left(\frac{s}{\omega_o} \right)^2$$

I will use this form for the illustration of Middlebrook's low entropy approach to analyzing second order circuits. The familiar Q (quality factor) represents how frequency selective the circuit is and ω_o can represent a resonance frequency or a reference frequency for the second order frequency response.

The familiar series RLC circuit configured as a low pass network, shown in **Figure 6**, can be easily analyzed to produce the transfer function

$$\frac{v_o}{v_i} = \frac{1/LC}{1/LC + \frac{R}{L}s + s^2} = \frac{1}{1 + \frac{1}{Q} \left(\frac{s}{\omega_o} \right) + \left(\frac{s}{\omega_o} \right)^2}$$

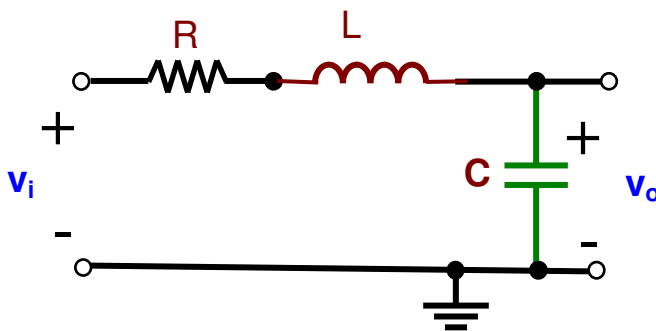


Figure 6 – Series RLC Circuit

where $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$ and $\omega_o = \frac{1}{\sqrt{LC}}$. Note that the denominator is a quadratic equation

based on the variable s . Solving this denominator for the roots will give us the two pole frequencies of the circuit. We also know that the value of Q determines whether the roots are real or complex, with the dividing line between the two being $Q = 0.5$.

The algebraic form of the quadratic equation that it seems almost everybody learned in school was $ax^2 + bx + c = 0$, with roots written as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

From this, one might assume that the roots depend on three constants. But this really isn't the case. The equation for the RLC circuit above indicates that they only depend on two constants, Q

and ω_o . To better understand this, I will briefly show how this works in general and how the result can be used to quickly estimate the frequency behavior of any second order circuit. For brevity's sake, I will keep this terse and just give the highlights.

First of all, a quadratic equation can always be rearranged to produce a leading 1, as is done for the *RLC* circuit above. A general form of this situation can be written as $1 + a_1s + a_2s^2$. Utilizing the familiar quadratic solution, the roots can be written as

$$s = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2a_2}$$

Some further algebraic work, with the relationship between Q and a_1 and a_2 easily demonstrable, looks like the following.

$$\text{Roots of this quadratic: } s = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2a_2}$$

$$s = -\frac{a_1}{a_2} \left(\frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4a_2/a_1^2} \right) = -\frac{a_1}{a_2} \left(\frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4Q^2} \right) \quad \left(Q = \frac{\sqrt{a_2}}{a_1} \right)$$

$$\text{So } s_1 = -\frac{a_1}{a_2} \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - 4Q^2} \right) \quad s_2 = -\frac{a_1}{a_2} \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - 4Q^2} \right)$$

$$\text{Define } F \equiv \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4Q^2} \quad \text{So } s_2 = -\frac{a_1}{a_2} F \quad s_1 = -\frac{a_1}{a_2} (?)$$

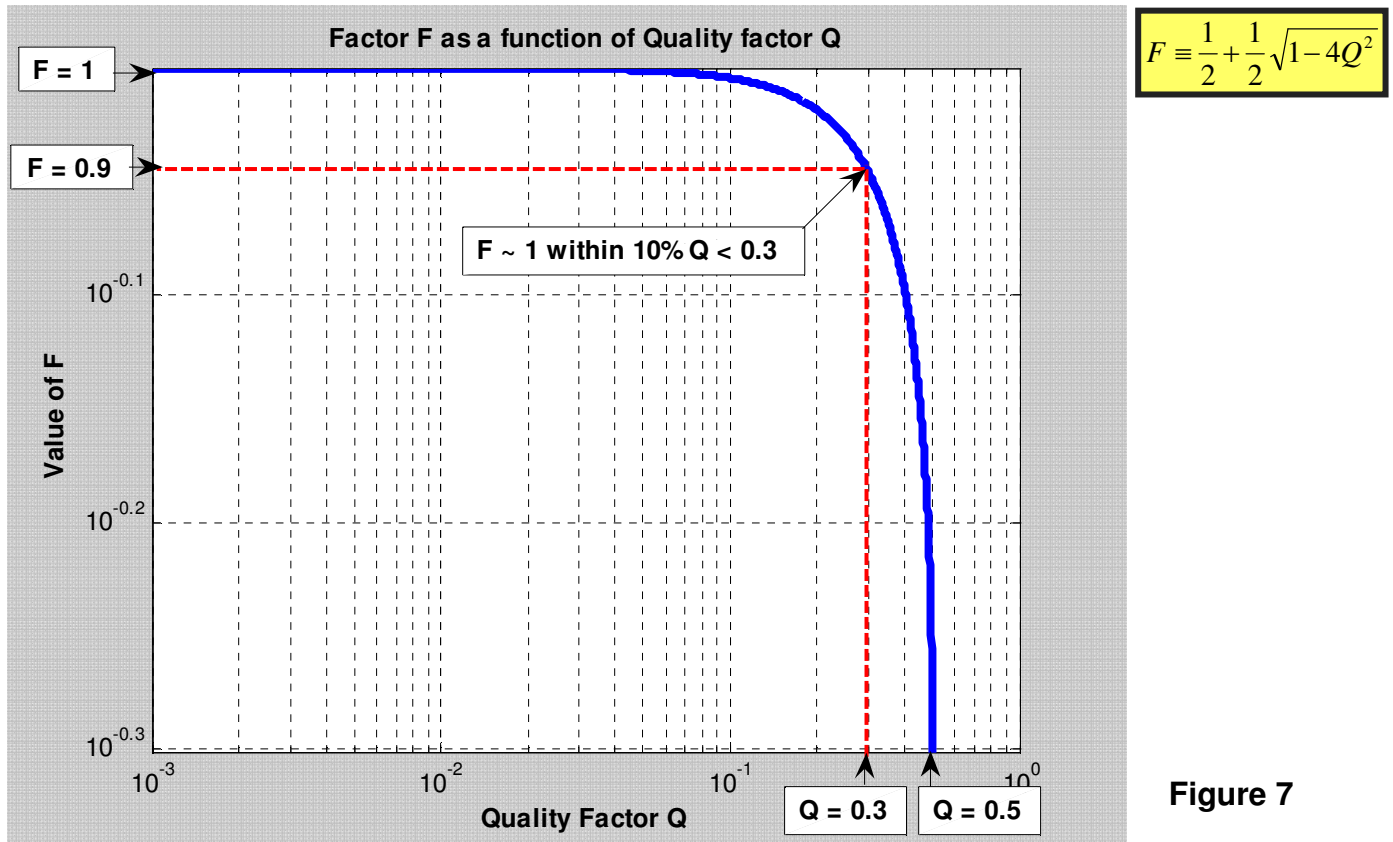
From this we can see that one root (s_2) is the ratio of the two constants times **a factor called F** which is a function of Q . When calculating the root s_1 , sometimes this can lead to taking a small difference of large numbers, which can lead to possible round off error. To avoid this, one other old algebraic 'trick' can be dusted off and applied – completing the square, as demonstrated below.

$$s_1 = \frac{-\frac{a_1}{a_2} \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - 4Q^2} \right) \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - 4Q^2} \right)}{\left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - 4Q^2} \right)} \quad (\text{complete the square})$$

$$s_1 = \frac{-\frac{a_1}{a_2} \left(\frac{1}{4} - \frac{1}{4} (1 - 4Q^2) \right)}{F} = -\frac{a_1}{a_2} \frac{Q^2}{F} \quad \text{and } s_2 = -\frac{a_1}{a_2} F$$

$$\text{NOTE: } \frac{s_1}{s_2} = \frac{-\frac{a_1}{a_2} \frac{Q^2}{F}}{-\frac{a_1}{a_2} F} = \frac{Q^2}{F^2} !!$$

This allows s_1 to be written in a form that avoids the round off error. Remember that both Q and F are functions of a_1 and a_2 . What kind of function is F ? If we graph it as a function of Q we get the following as rendered in MATLAB® in **Figure 7**.



Note that when Q is less than 0.3, F can be approximated as $=1$ within 10% error. This means for circuits that are overdamped ($Q < 0.5$), F can be replaced with 1 with only a small error. If Q is greater than 0.3, then the exact F value probably should be used, but by including F in either case we can get an exact answer. If we use the $F \approx 1$ approximation, the two roots reduce to

$$s_2 = -\frac{a_1}{a_2} F \approx -\frac{a_1}{a_2} \quad \text{and} \quad s_1 = -\frac{a_1 Q^2}{a_2 F} \approx -\frac{1}{a_1} \cdot \text{Simple!}$$

Another useful result for determining pole frequencies for a quadratic circuit is that the pole frequencies can also be written as

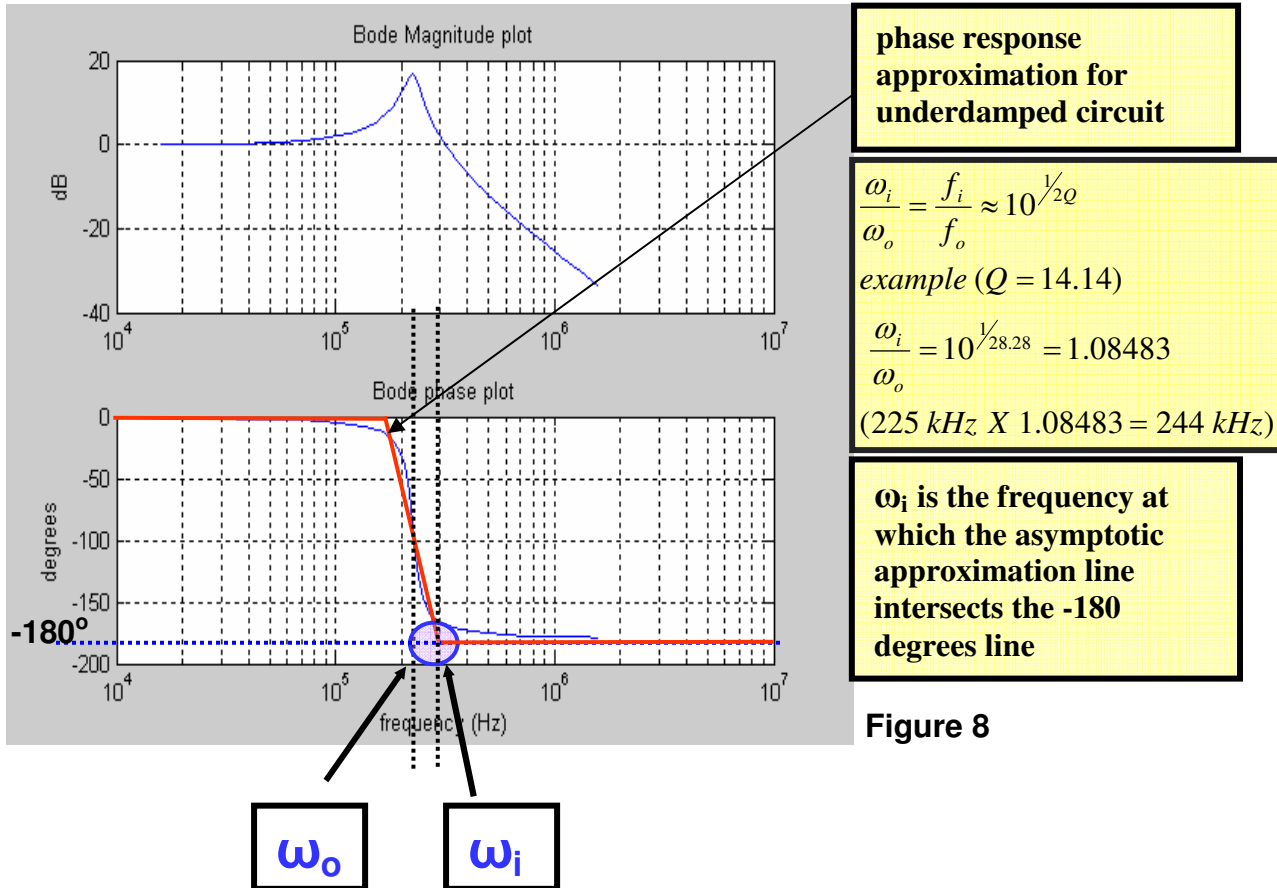
$$s_1 = -\frac{a_1 Q^2}{a_2 F} = -\frac{1}{\sqrt{a_2}} \frac{a_1 Q^2}{\sqrt{a_2} F} = -\frac{\omega_o Q^2}{Q F} = \frac{-\omega_o Q}{F}$$

$$s_2 = -\frac{a_1}{a_2} F = -\frac{1}{\sqrt{a_2}} \frac{a_1 F}{\sqrt{a_2}} = -\frac{\omega_o}{Q} F$$

If $F \approx 1$, then the two roots are simply the *product* of Q and ω_o , and their *quotient*. This is very useful because it relates the poles to behavioral quantities that may be part of a specification. Using this approach and guided by asymptotic approximations, one can quickly

sketch out the expected Bode plots for any overdamped circuit. Have you seen this in any textbooks?

Middlebrook has also applied his D-OA paradigm of seeking ways to make analysis more useful for design to the case of complex roots. Starting with the corner frequency ω_o and the value of Q , he shows how the Bode behavior can be quickly estimated. The most unique aspect of this work is that he shows how not only the magnitude, but also the phase response can be estimated for the complex roots case, as shown in **Figure 8** below. The factor $10^{1/2Q}$ is the result of a least-squares fit analysis.¹⁵



These ideas can be extended to other than passive circuits. Transistors are often modeled in textbooks with two internal capacitors. In the case of the FET they are the gate-to-source capacitance C_{gs} and the gate-to-drain capacitance C_{gd} . Typically Miller's theorem is employed to convert the second order reality of this circuit to a first order approximation, which is based on an assumption of a dominant pole.¹² This is often the case, but what is missing is a complete understanding of the frequency behavior of the circuit under all conditions, including when a dominant pole may *not* be present. Application of Middlebrook's method of improve quadratic roots allows the student more complete understanding what is going on and allows for an efficient way to get *exact* pole frequency values under any condition!

Armed with this approach to the analysis, it is possible to create a systematic table similar to that shown for first order circuits in **Figure 5**, for the asymptotic approximations of quadratic circuit responses. With the first order case, the problem can be reduced to finding two constants (K and τ), whereas with the second order case it can be reduced to finding three constants (Q , ω_o , and K), where the all-important constant Q defines a major divide between real and complex poles. Since Q determines the amount of peaking in the response, this adds an additional item to be determined as compared to the first order case. However, once just one of these responses has been determined for a given type of response, e.g. the low pass configuration, then the other three are easily determined from it.

The Extra Element Theorem

A circuit containing reactive components does not have to be very complex before the algebra of traditional frequency analysis methods get out of hand. For example, the circuit shown in **Figure 9**, which could represent the input channel of an oscilloscope (R_2 and C_2) in series with a compensated oscilloscope probe (R_1 and C_1). If we were to utilize the impedance values for the capacitors ($1/Cs$) and use a voltage divider approach to finding the transfer function, we would get the following.

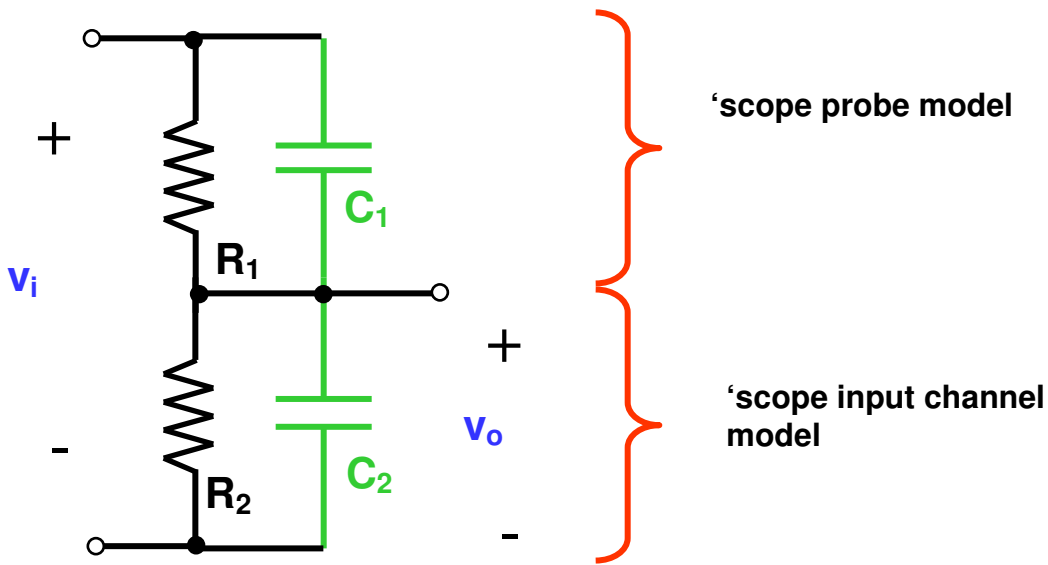


Figure 9

$$\frac{v_o}{v_i} = \frac{\frac{R_2}{R_2 C_2 s + 1}}{\frac{R_2}{R_2 C_2 s + 1} + \frac{R_1}{R_1 C_1 s + 1}} = \frac{R_2}{R_2 + R_1 \frac{(R_2 C_2 s + 1)}{(R_1 C_1 s + 1)}}$$

What useful information can be gleaned from this form? First, we can recognize that when the time constants are equal ($R_2 C_2 = R_1 C_1$) then the circuit is frequency independent and attenuates the input by a voltage divider factor. But what happens when the time constants are not equal? What does the frequency behavior look like? What difference is there when one time constant is greater than the other? We have no specific information. Even though this equation doesn't

look particularly disordered, it still is a high entropy form as far as usefulness for analysis and design is concerned. Where are the poles and (possible) zeros? It is not clear.

We could come up with the time constant by setting the input to zero and noting that we end up with the parallel combination of the two resistors in parallel and the two capacitors in parallel. That would mean that the time constant would be $\tau = (R_1 \parallel R_2)(C_1 + C_2)$. However, we did not get this information from the transfer function! We really need a low entropy form – a form that will give us the maximum amount of information about the circuit. It turns out that by successive application of Thevenin’s Theorem and considerable effort, it is possible to derive the

more useful form of the transfer function $\frac{v_o}{v_i} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{(1 + s/\omega_z)}{1 + s/\omega_p}$ where the pole frequency ω_p and the zero frequency ω_z are defined as

$$\omega_p = \frac{1}{(R_1 \parallel R_2)(C_1 + C_2)} = \frac{C_1}{C_1 + C_2} \frac{1}{R_1 C_1} + \frac{C_2}{C_1 + C_2} \frac{1}{R_2 C_2} \quad \text{and} \quad \omega_z = \frac{1}{R_1 C_1}.$$

However, with a theorem developed by Middlebrook and thoroughly explained in his course called the **Extra Element Theorem**, or EET, this low entropy result can be derived with much less work and less susceptibility to error. A complete development of this is beyond the scope of this paper, but the basic concept is to start with a simpler circuit, in this case let’s use the circuit with C_1 removed and first determine the transfer function for that. In this example the transfer function has been already determined in this paper in first example (see **Figure 2**) as

$$\frac{v_o(s)}{v_i(s)} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{1}{(1 + s/\omega_o)} \quad \text{Where in this case } \omega_o \text{ would be } \frac{1}{(R_1 \parallel R_2)C_2}.$$

According to the EET, this transfer function would be called A_∞ or the gain when the impedance of the extra element C_1 is infinite. Then, the transfer function for the circuit with the extra element included would take the form

$$\frac{v_o(s)}{v_i(s)} = A = A_\infty \frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}}$$

Where Z is the impedance of the extra element C_1 , and Z_d and Z_n are impedances defined by Middlebrook as the **driving point impedance** and the null double injection impedance or just **null impedance**. The concept of null double injection is a new concept to most of us although it has appeared in publication.⁶ The driving point impedance Z_d turns out to be the impedance seen looking into the point here C_1 is attached (the driving point) when the input voltage is zero. **Figure 10** illustrates this situation. From the circuit diagram we can see that Z_d will be $(R_1 \parallel R_2) \parallel (1/(C_2 s))$, which can be written as

$$Z_d = \frac{(R_1 \parallel R_2)}{1 + (R_1 \parallel R_2)C_2s}$$

The null impedance is defined to be the impedance seen by a source at the driving point when: (1) the input signal is restored, (2) a second source is assumed to be attached at the driving point, and (3) the two sources are mutually adjusted so that the defined output voltage is zero (nulled). This condition is called Null Double Injection. This case is illustrated in **Figure 11**. Since the output is zero, that means no current flows through $R_2 \parallel C_2$. This fact dictates that there is no current flowing through the input source, which implies that the current flowing through the driving point source flows only through R_1 . Consequently, the impedance seen by the driving point source is simply R_1 . This is the null impedance Z_n . By inserting the expressions for A_∞ , Z_d , and Z_n , one can quickly formulate the low entropy result shown above in yellow as shown below.

$$\begin{aligned} \left[\frac{R_2}{R_2 + R_1} \right] \frac{1}{\left(1 + \frac{s}{\omega_p}\right)} \frac{1 + \frac{Z_n}{Z_{C1}}}{1 + \frac{Z_d}{Z_{C1}}} &= \left[\frac{R_2}{R_2 + R_1} \right] \frac{1}{\cancel{(1 + (R_1 \parallel R_2)C_2s)}} \frac{1 + R_1C_1s}{1 + \frac{(R_1 \parallel R_2)C_1s}{\cancel{1 + (R_1 \parallel R_2)C_2s}}} \\ &= \left[\frac{R_2}{R_2 + R_1} \right] \frac{1 + R_1C_1s}{1 + (R_1 \parallel R_2)C_2s + (R_1 \parallel R_2)C_1s} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{1 + R_1C_1s}{1 + (R_1 \parallel R_2)(C_2 + C_1)s} \\ &= \frac{v_o}{v_i} = \left[\frac{R_2}{R_2 + R_1} \right] \frac{(1 + s/\omega_z)}{1 + s/\omega_p} \quad \text{where} \quad \omega_z = \frac{1}{R_1C} \quad \omega_p = \frac{1}{(R_1 \parallel R_2)(C_2 + C_1)} \end{aligned}$$

This form can not only tell you, with a small amount of algebra, that the attenuation is frequency invariant, but it also allows you to quantitatively know the frequency behavior for any set of element values! It also explicitly shows that the response is characterized by a voltage divider, one zero, and one pole.

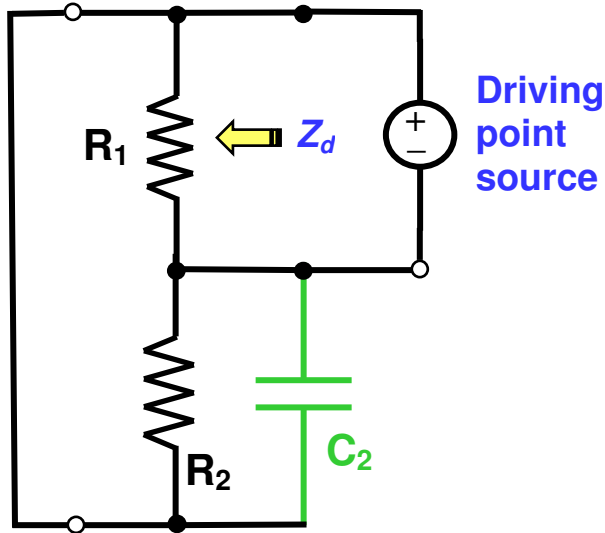


Figure 10

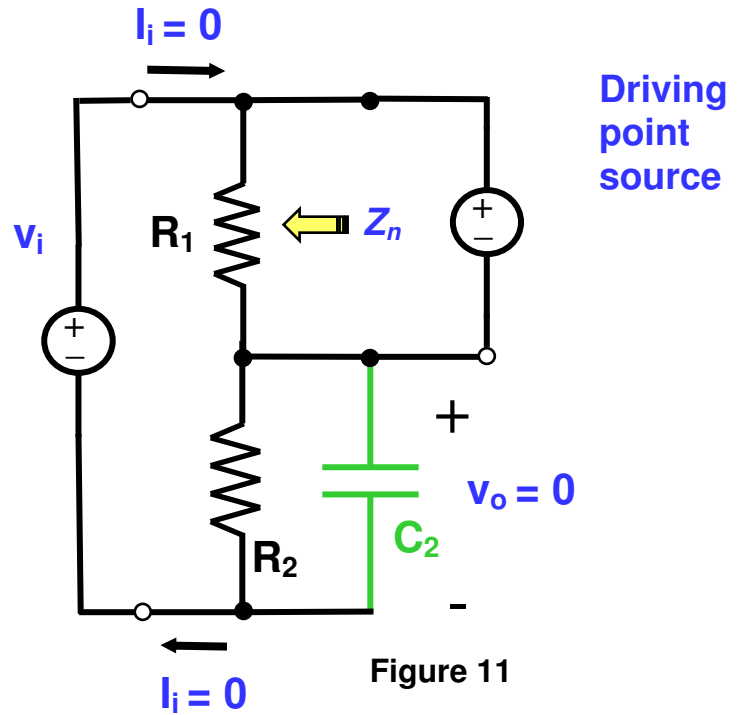


Figure 11

Application of the EET and the D-OA Mindset to Electronic Amplifiers

This paper started out with the proposition that educators should learn about and incorporate the practical and available design-oriented techniques developed over several decades by now-emeritus Caltech Professor R. David Middlebrook. There is much more material than can possibly be given a respectful airing in a format such as this paper; however, I would like to include just one more example taken from a textbook and show how the solution given there contrasts with the low entropy alternative developed from the application of D-OA concepts. The example is a single stage common source FET amplifier as shown in **Figure 12** on the next page, taken largely from an electronics textbook.¹² It applies Miller's Theorem to the high frequency part of the response and, separately, standard analysis to the low frequency part. Combining both of these parts yields

$$\frac{V_o}{V_{sig}} = - \left(\frac{R_{in}}{R_{in} + R_{sig}} \right) \left[g_m (R_D \parallel r_o \parallel R_L) \right] \left(\frac{s}{s + \omega_{p1}} \right) \left(\frac{s + \omega_Z}{s + \omega_{p2}} \right) \left(\frac{s}{s + \omega_{p3}} \right) \left(\frac{1}{1 + s / \omega_H} \right)$$

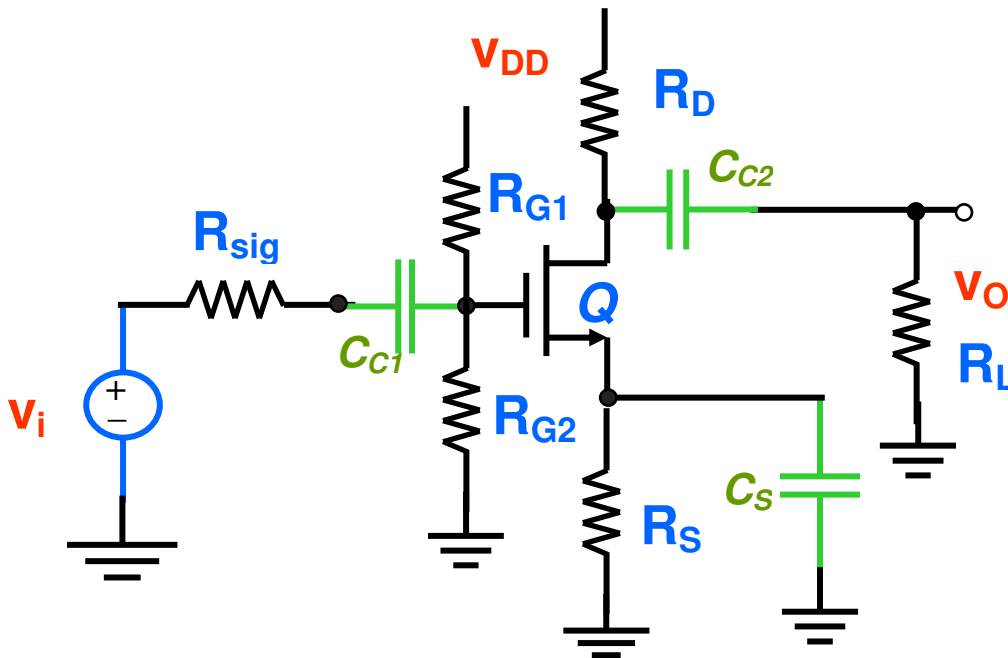
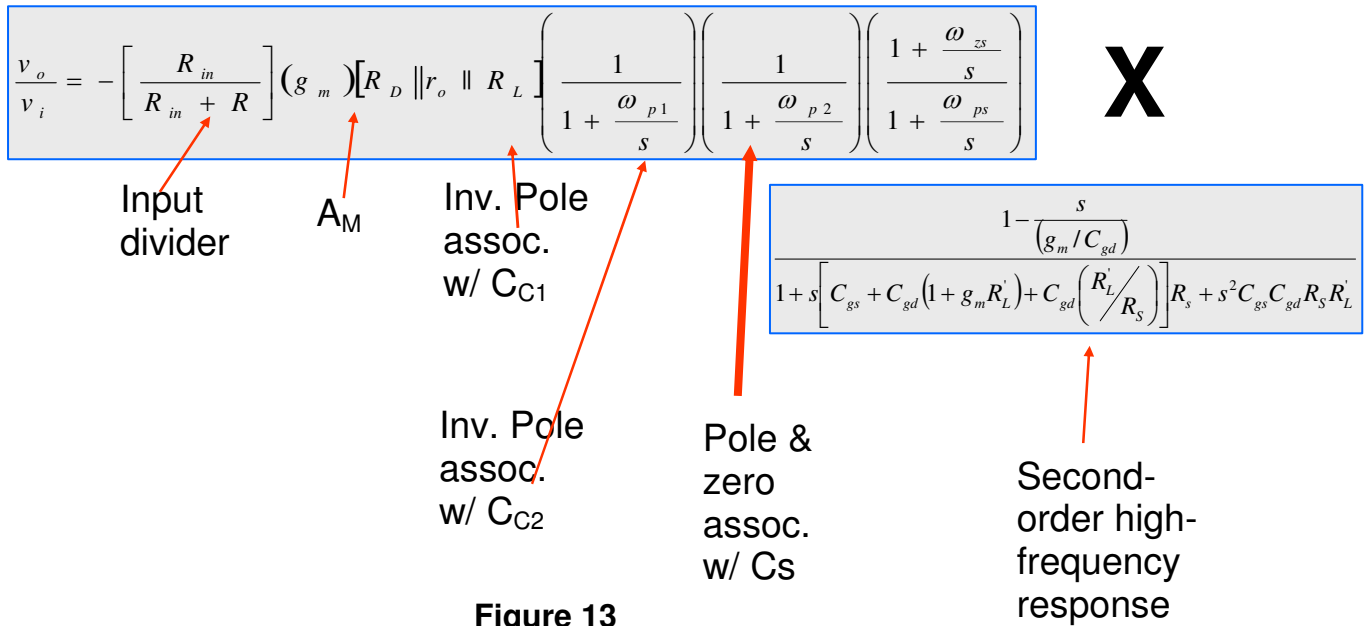


Figure 12

The very last term is the high frequency term where the second order behavior of the FET has been reduced to a single pole. The other three terms relate to the low frequency behavior associated with the two coupling capacitors and the bypass capacitor C_S . $R_{in} = R_{G1} \parallel R_{G2}$. So in this rendition, we have lost the exact quadratic information and the low frequency behavior is shown as normal poles combined with three zeros at zero frequency. In contrast, the application of D-OA based low entropy thinking, including the EET, yields what is shown in **Figure 13**. The indicated physical interpretation of the factors is included. Note that this form has explicit factors representing inverted poles and one inverted zero. They are inverted because the reference frequency range is midband, where the amplifier's amplitude is flat.

The quadratic may appear complex, yet the same technique that was illustrated with the series RLC circuit can be applied just as well in this case, i.e., the denominator is in a form that contains a leading term of unity, so the Q and ω_o quantities can be quickly related to component values. Then, the value of Q can be determined and then the complete second order behavior worked out as before. This approach allows the retention of complete frequency information and an efficient, less error-prone analysis procedure. This is empowering rather than defeating or discouraging. The complexity is under control and maximum design related information is developed.

What does not show up explicitly when comparing the two expressions is how much less work it takes to obtain the low entropy version via an application of the EET. In a fashion similar to the example of the oscilloscope channel/probe circuit, the effort and complexity of deriving the effect of the bypass capacitor C_S on the transfer function is significantly reduced.



Conclusion

Middlebrook’s circuit analysis techniques do not just represent a useful set of ‘tricks’ to make things easier for the designer, but rather they embody a different way of thinking about how to do analysis. They involve an intentional strategy of keeping the entropy low throughout the analysis process so things never get out of hand algebraically and the physical interpretation of what the circuit expressions mean is always kept in view. I contend that analysis methods presented in our textbooks trend toward high entropy and thus lead to mathematical paralysis, especially when they are applied to real world problems requiring real world answers. I am not saying that our textbooks have it all wrong. They assuredly *have* content that is compatible with the low entropy concept; however, my argument is that *we can do better* and *need to do better*.

It is one thing to make the philosophical case for keeping the complexity, or entropy, under control. Few would object to this idea. However, it is yet an entirely different thing to provide practical tools for actually doing it! This is exactly what Dr. Middlebrook has done with his cohesive set of theorems and other techniques whose practical usefulness have already been established but need wider adoption by educators.

I will conclude this paper with the last phrase of the title: *why aren’t we using it to teach our students?*

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