What is the Derivative of Music?

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What is the Derivative of Music?

Abstract

In our continuing effort to prove to students that *Signals & Systems* is not just another mathematics course taught by the ECE Department, we ask the question, "What is the Derivative of Music?"

The first-order difference (or first-difference) is an incredibly simple algorithm that very accurately approximates the numeric derivative operator, especially for oversampled signals. Its inverse also accurately approximates the numeric integration operator, but not without numeric difficulty.

Given a real-time demonstration using winDSK8, we can now show students that these mathematical operators provide powerful signal processing filtering tools for real-world signals.

During this ASEE session, we will include the demonstration that the derivative operator is more than a symbolic mathematical operator and much more than just another academic exercise.

We have successfully used winDSK, winDSK6, and the latest version, winDSK8, to provide demonstrations of any number of concepts during outreach (K-12 events), at freshman motivational events, and in junior, senior, and even graduate ECE courses.

Introduction

For years, students have struggled with learning the significance of the impulse response. This is especially true given that there is no piece of test and measurement equipment (T&ME) in our teaching laboratories that is capable of producing a true impulse. Many educators have written about the benefits of demonstrations to aid student learning, especially for some of these more difficult topics [1-15].

The discrete-time equivalent of the impulse response, the unit sample response, can be just as onerous for students to understand, since it's regularly viewed as "just a computer simulation" and not related to anything that's practical or happening in the real-world.

To illustrate this point, for several years, the authors have presented to students a question similar to, "In your own words, define the term, *impulse response*." The average score on this question was routinely the lowest of any of the questions on the *Signals & Systems* final examination. This improved significantly when real-time demonstrations, other hardware demonstrations, and laboratory exercises were introduced in the class.

Background

During outreach (K-12 events), at freshman motivational events, and in junior, senior, and even graduate ECE courses, we have used real-time systems to demonstrate the utility of user-programmable devices.

During the freshman or early ECE course events, where the derivative operator is (hopefully) well understood, we ask the question, “What is the Derivative of Music?” The typical student response is, “Give me the equation of the music and I can calculate its derivative.” Our reply is usually along the lines of, “If you only want to listen to sinusoids or other simple periodic waveforms, I can give you an equation. But can *you* write the equation for your favorite song?”

This allows a seamless transition to the fact that while not all signals are *born digital*, we regularly process our signals in their digital or discrete form.

Returning our discussion to the derivative operator, a student who has learned the fundamentals of the Laplace transform should recognize the derivative, d/dt as the Laplace independent variable s. In most texts, $s = \sigma + j\omega$, and for a discussion of the frequency response, setting $\sigma = 0$ results in the classic result, $s = j\omega$. At this point, a *Signals & Systems* student should be able to recreate Figure 1. This should be recognized as the magnitude only display of a Bode plot for the response of an ideal integrator, 1/s, and an ideal differentiator, s.

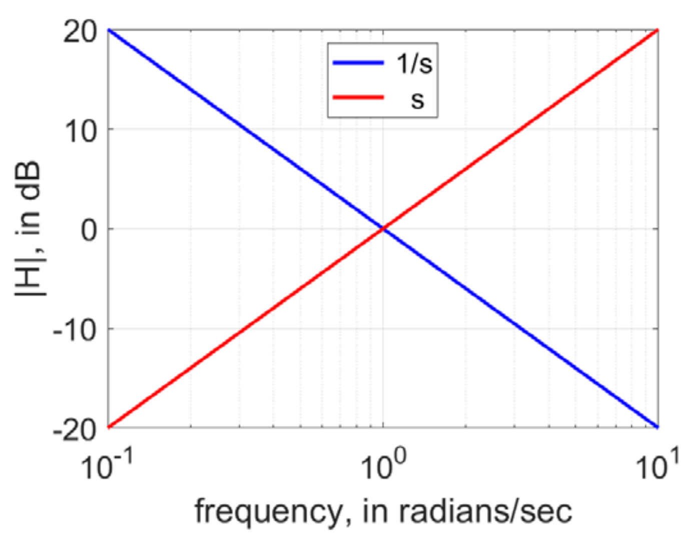

Figure 1. The frequency response (magnitude only) of an ideal integrator, 1/s, and an ideal differentiator, s. This is a portion of a Bode plot; phase is not shown.

In the sampled world, the result of the first-order difference, y[n] = x[n] – x[n – 1], is shown in Figure 2. The first-order difference or first difference, is just the difference between the current and previous sample values. Figure 2 doesn't look much like the plot of a differentiator that we saw in Figure 1. But after shifting to logarithmic axes, shown in Figure 3, this now looks very similar to the associated plot in Figure 1. This provides a wonderful opportunity to discuss with students a number of related topics, including sampling, aliasing, use of the decibel (dB), and filtering.

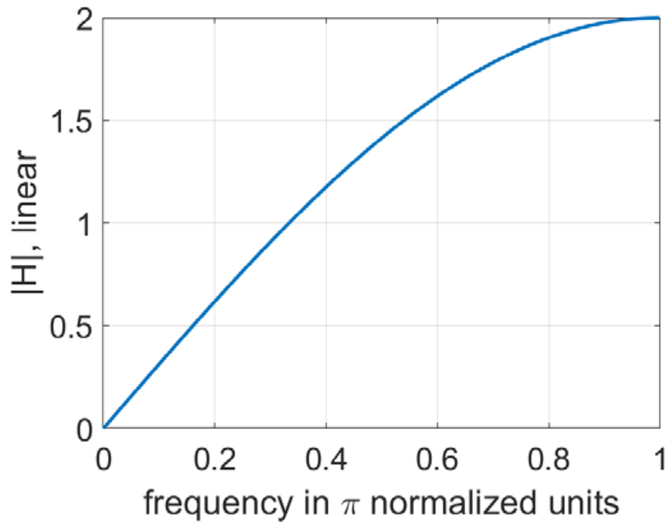

Figure 2. The frequency response (magnitude only) of the first-order difference, using linear axes.

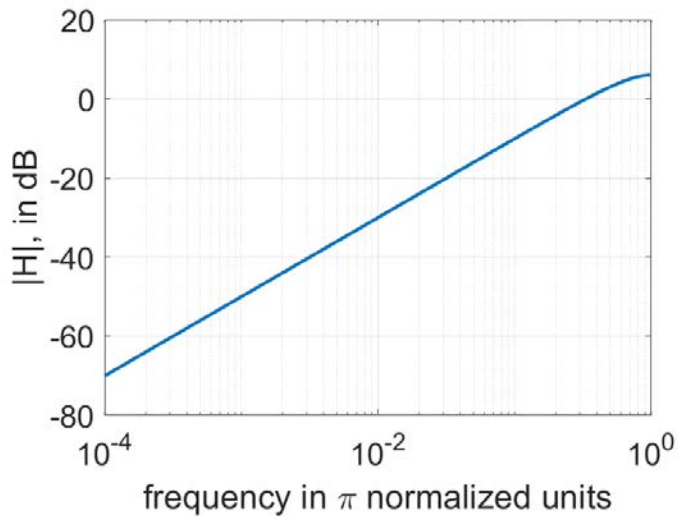

Figure 3. The frequency response (magnitude only) of the first-order difference, using logarithmic axes.

In MATLAB, the difference operator is implemented using the **diff** command. MATLAB can also calculate higher-order differences using this same **diff** command.

While s plots as a perfectly straight line in Figure 1, the first-order difference is incredibly close to a straight line for oversampled signals and only diverges from a straight line near the very upper end of the frequency axis, approaching the maximum frequency, Fs/2. It is easy to shift the magnitude value up or down, by simply applying a scale factor or gain. Similar observations can be made regarding the discrete-time version of 1/s, as shown in Figure 4.

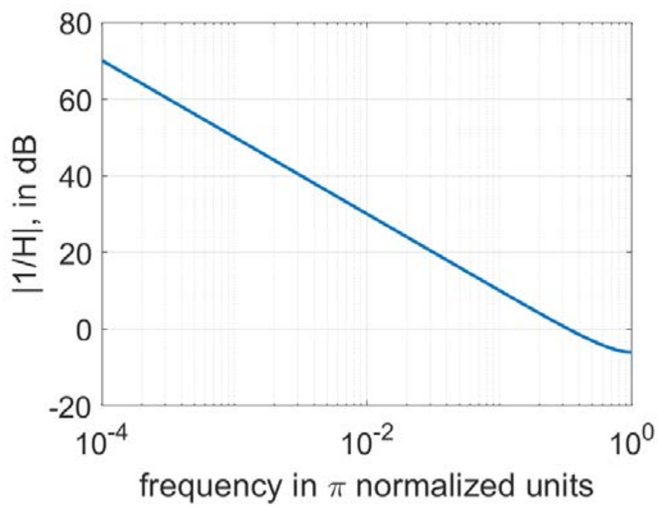

Figure 4. The frequency response (magnitude only) of the reciprocal of the first-order difference, using logarithmic axes.

Now that the frequency responses for s and 1/s (i.e., the first-order difference and its reciprocal) have been determined, the straightforward observation can be made that these actually represent filters. Specifically, s and the first-order difference are highpass filters and 1/s, the reciprocal of the first-order difference, are lowpass filters.

It must be noted that the pure reciprocal of the first-order difference becomes y[n] = y[n-1] + x[n], which is actually an ideal accumulator, and this form will cause stability issues.

This observation then leads naturally to a discussion with students of what is sometimes called the *leaky integrator*, y[n] = gain*y[n-1] + x[n], where gain is a value between 0 and 1, but usually very close to 1, e.g., 0.99. The gain value pulls the system's pole off of the unit circle, reducing the DC gain to a finite value, and resulting in a stable system. All of these topics can be stimulated by the simple question, "What is the Derivative of Music?"

Real-time demonstrations

The discussions with students are all accompanied by real-time demonstrations using winDSK8. This program, designed specifically for educators, and freely available to educators, has evolved and improved over the years [11-16]. A complete description of what is needed and how to connect both the hardware and software necessary to perform these and other demonstrations is provided in [16].

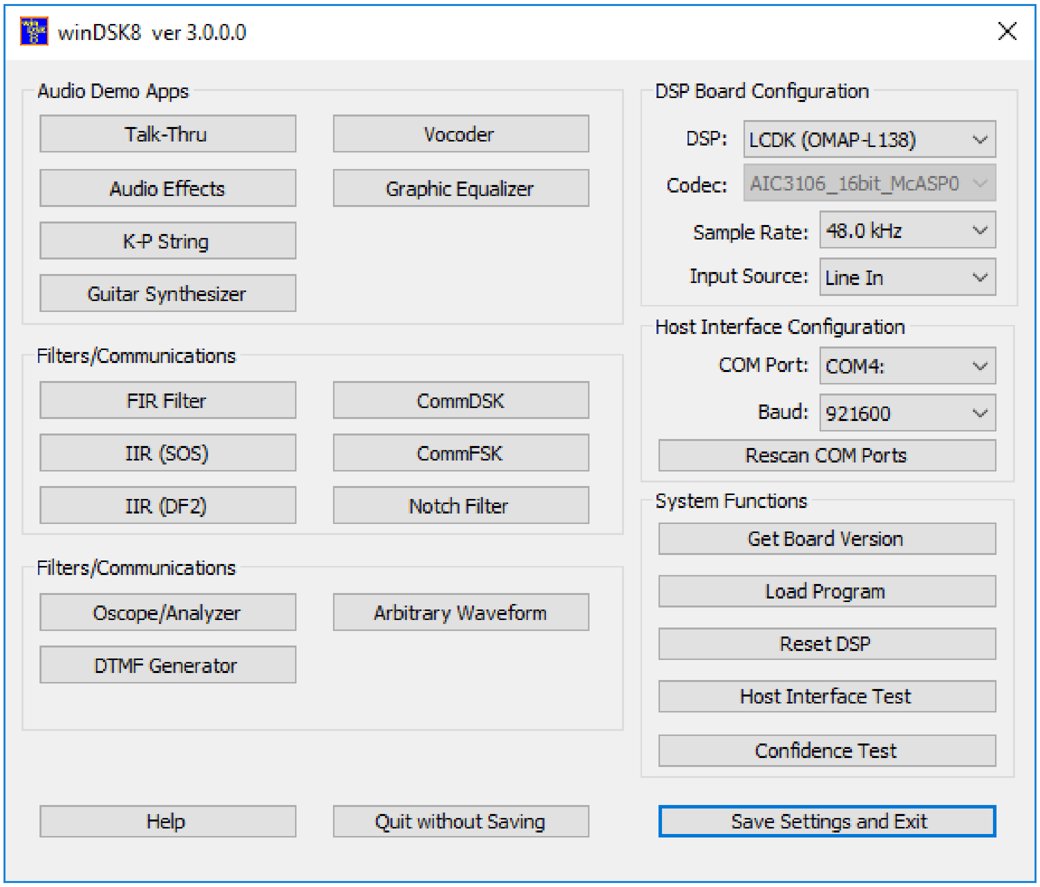

Figure 5. The winDSK8 program main graphical user interface screen.

winDSK8 is designed to work with the TI LCDK (low cost development board). The GUI (graphical user interface) for winDSK8 allows for hundreds of different demonstrations, including everything that we have discussed in this paper. For the topics we are discussing here, the ability to quickly implement and demonstrate both FIR (shown in Figure 6) and IIR (shown in Figure 7) digital filters is a major benefit for the professor and the students.

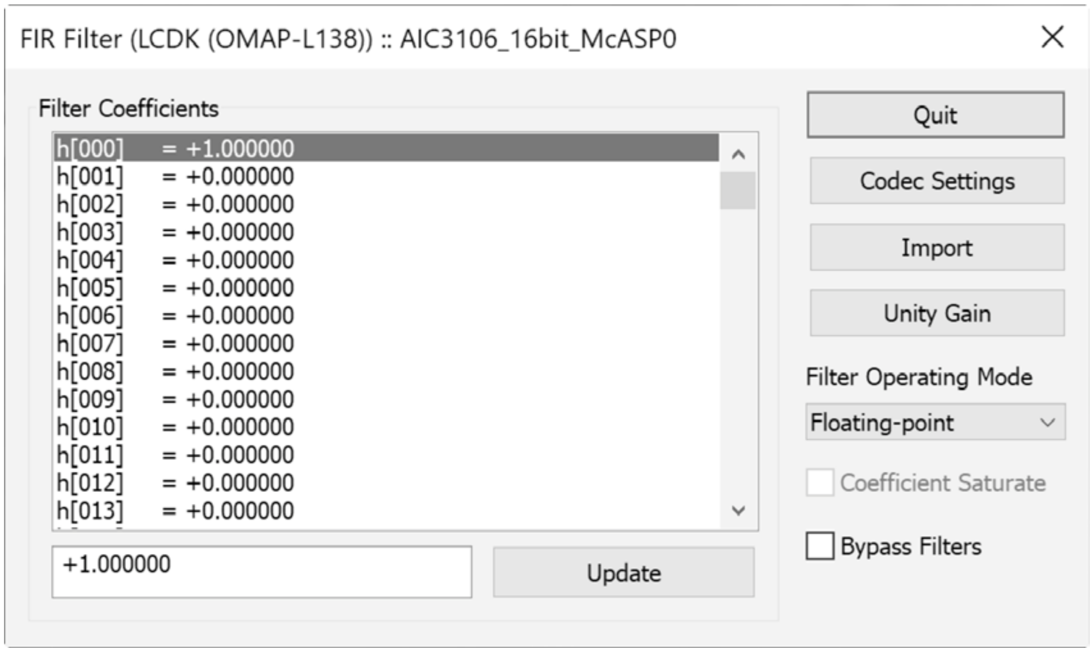

Figure 6. The winDSK8 program graphical user interface screen for defining and demonstrating FIR digital filters.

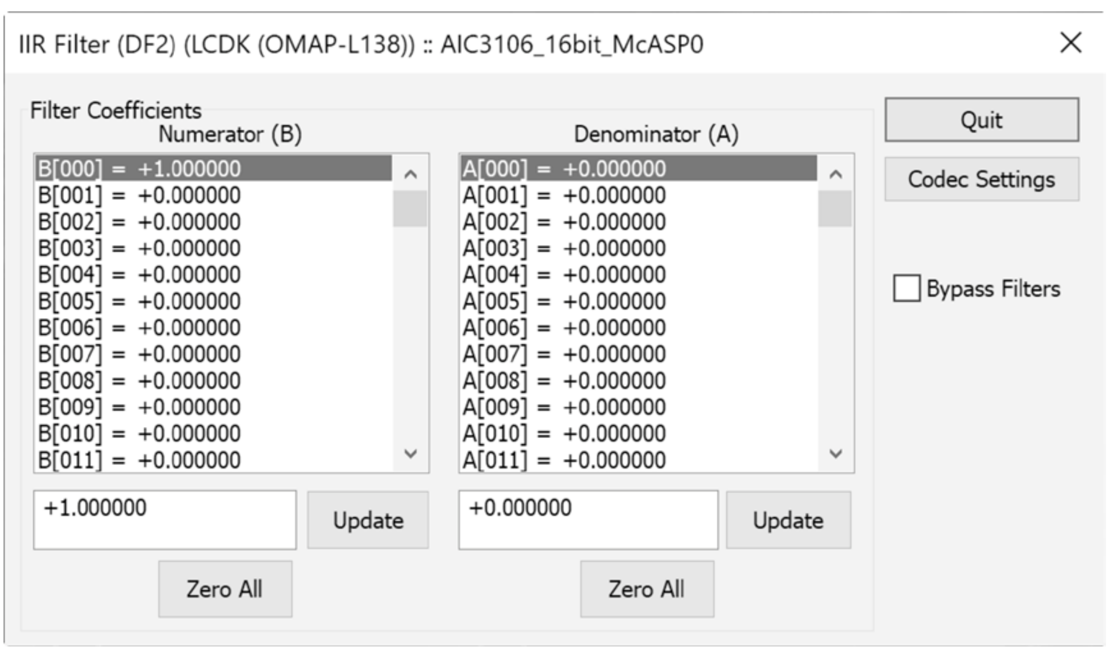

Figure 7. The winDSK8 program graphical user interface screen for defining and demonstrating IIR digital filters.

Conclusions

After this quick demonstration, all of the current *Signals & Systems* students were correctly able to identify the derivative of music as a highpass filtered version of the original music signal. Similar success was demonstrated with the integrator, a lowpass filter, and with varying the magnitude of the unit impulse response, a volume control.

Real-world signals and the real-time processing of these signals, especially using student-provided music for the signals, is remarkably motivational to our students. It has led to a much better understanding of what had been previously viewed by almost all *Signals & Systems* students as "just another math class."

During a requested guest lecture opportunity during this past academic year, 24 of 25 students felt that this demonstration helped them better understand *Signals & Systems* and all students wanted additional demonstrations and applications to be included in the class.

We encourage all professors to consider using real-world signals in all of their classes. The authors have repeatedly offered a three-credit, semester long, elective course in real-time signal processing. While our preference is to process these signals in real-time, offline processing can be almost as effective.

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