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Where's the Math in Engineering?

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Abstract

Engineering analysis depends on modeling the physical world mathematically. In engineering practice, the models are often already developed, the solutions derived, and a computer program written to carry out the calculations. Practicing engineers are engaged in applying the answers; model development and the computer programming required to generate the answers is literally bought and otherwise, largely ignored. Has engineering analysis really become an Input/Output process devoid of mathematics? In teaching students to be engineers, instructors strive to develop a set of reasoning and mathematical skills that apparently are "never used".

In this paper, perceived student shortcomings that inhibit a student's acceptance, development, and lifelong use of mathematics are discussed and several interventions are proposed. The paper concludes with a discussion of soil consolidation. The discussion points out the need to introduce modeling concepts and models of fundamental physical phenomena early on in engineering coursework and to then continuously use those mathematical models to describe increasingly sophisticated processes. The goal is to continuously develop and apply knowledge of mathematics while in school and throughout professional practice.

Introduction

Engineering analysis depends on mathematical models of the physical world; however, in engineering practice, the models are often already developed, the solutions derived, and a computer program written to carry out the calculations. Practicing engineers are engaged in applying the answers; the model development and computer programming required to generate those answers is purchased and usually ignored. Practicing engineers, doing work for hire, can't afford to reinvent published solutions. With this in mind, some might conclude that engineering analysis is simply an I/O process where engineers match the task to a known solution path. In teaching students to become engineers, instructors strive to develop critical thinking, knowledge, and skills in mathematics and modeling that are apparently "never used". In our fast-paced and highly competitive world, has it become too expensive to think?

Recently, Dr. Brooks (a mathematics professor) decided to take an engineering course (Statics, taught by Dr. McDonald) for credit to see firsthand how Calculus I, a prerequisite to Statics, is applied. That semester, most of the two dozen Statics students, Dr. Brooks' classmates, were simultaneously Dr. Brooks' Calculus II students in the afternoon. Dr. Brooks' intent in taking Statics was to tailor future math instruction to engineering education. The result, for both instructors, was a realization of how different the learning environments are in mathematics and engineering. Both developed an awareness of a need to change the way math is taught and used.

The Western Illinois University Quad Cities campus in Moline, Illinois is a branch of the main campus located in Macomb, Illinois. The Quad Cities campus serves approximately 1,500 students, of which 10% are engineering majors. Dr. Brooks is the only full-time math instructor at the Quad Cities campus, and math enrollment, particularly in Calculus and Differential Equations, is composed almost entirely of engineering students. Statics is the first engineering course taken by engineering majors, and although it is numbered as a 200-level course, it is taught to freshman in the second semester. The size of the engineering program (approximately 150 students) virtually ensures that student cohorts (one per year) form as they work through the program; typically, only one section of each course is offered each year.

Mathematics is a world of rules, rigor, and steps. The beauty of math is that once the rules are mastered, they never change. Math is often learned by repetition; given a particular type of problem, students follow a procedure to arrive at the desired outcome. Many students pass their math courses using memorization or rote learning. [1] Conversely, engineering problems require students to determine the most appropriate set of rules to follow in order to achieve a desired outcome. This may require a higher level of learning – a level in which students are often poorly prepared to engage. [2] The reality is that in engineering, mathematics is a convenience to be abused; some of the rules can be overlooked, so long as the effect on the outcome is insignificant or at the least, tolerable. A pure mathematician is appalled by the liberties taken by engineers. To expertly utilize the convenience of math requires that students 'know' the math rather than just memorize a set of rules.

As students transition from math and science classrooms into engineering courses, a transformation should occur. Much as a caterpillar encases itself in a cocoon to later emerge as a butterfly, students well-versed in math need to work through the introductory engineering courses by learning how to apply and use their math knowledge. During the transition, they should be learning that it's not enough to simply answer a question. They increasingly need to explain, justify, and defend their solutions. Why? Because as they advance, there is no longer a single answer or simply one way to solve a problem. As the problems become more involved, so do the solutions, and there is usually more than one way to obtain a solution. The best way is subject to interpretation and/or circumstance; therefore, the solution path requires a degree of explanation and/or justification for the reader to easily follow the process.

Student Shortcomings

Mathematics is an ancient discipline dating back to before the early Greek and Babylonian dynasties. Although math has been studied for centuries, there is great hesitation from students when it comes to utilizing their skills outside of the math classrooms. From a mathematical perspective, one way to explain this is that students are severely lacking in critical thinking skills. As Stevenson and Stigler put it, "In mathematics, the weakness is not limited to inadequate mastery of routine operations, but reflects a poor understanding of how to use mathematics in solving meaningful problems." [3] Is this a result of adopting procedural approaches to problem solving and relying on rote memorization rather than conceptual understanding? [1] Does the lack of applied problem solving within mathematics courses affect students' ability to recognize the worth of mathematical concepts in outside disciplines? The result of these shortcomings may be a lack of deep understanding of the mechanics of mathematics. This, along with a general sense of disdain for the entire subject, makes it easy for students to leave the mathematics behind… if instructors allow it.

Prior to taking university-level engineering, math, and science courses, students follow a well-worn problem-solving path – a yellow brick road: if you see this, you do that. It is what today's students are trained in, used to, and accomplished at; they are rote learners. They believe they are studying and have achieved proficiency when they can repeat what was presented. In walking the yellow brick road, students avoid developing many important skills necessary for success in today's professional world.

Dr. Rabchuk, a Western Illinois University physics professor, says, "Learning math is like learning Spanish, while learning physics is like being dropped off in Mexico." In engineering, there is no less urgency in the learning process; however, now you're looking for a restroom!

As students transition from mathematics to engineering courses, they need to be applying their math skills continuously. One critical use is in the development of mathematical models. The equations used in engineering analysis and design don't just appear with the turn of a page. They were developed by someone for a specific case, and engineers should be able to not just repeat the standard development but employ a logical process that allows them to confidently solve similar situations. In the classroom, it is easy to skip the development of a model. Students don't want to develop models; they want to calculate answers. They want an equation they can use to calculate a value. If allowed to do this, as engineers they won't be able to do anything more than grind equations and post answers. They will have lost the ability to derive equations and with that, the ability to solve problems outside of the typical, mainstream, or mundane but easy work. They may even lose the ability to recognize the limitations of the published equations.

When asked about math, many practicing engineers will admit that they don't regularly use anything more than some trigonometry and a little algebra. Ask an engineer what they use, or query the internet for "what math do engineers really use". The query results in pages of comments from practicing engineers, and it is evident that many haven't ever used what amounts to obligatory math posted on their college transcript. ¿Dónde está el baño?

Means of Interventions

What can we, as instructors of mathematics or engineering, do to help our students? If students don't arrive with the skills required to succeed, most will not magically obtain them in a 'business as usual' environment. At some point, students must be led off the yellow brick road and onto paths that are more adventurous. Although these paths may be more arduous, they will almost certainly be more rewarding. Students probably won't be comfortable leaving the familiar and simple education model: rote learning. However, they often adjust quickly and soon realize that they have been missing out on a lot of... education.

Prior to enrolling in their first formal engineering course, students work their way through fundamental prerequisite courses such as calculus, chemistry, and physics. It is in these introductory courses that they are first introduced to mathematics as a means of describing and solving real-world problems. While math blends organically into science courses, the converse is not necessarily true. Mathematics instructors need to be intentional about incorporating problems that apply the concepts being conveyed to the disciplines they serve. Not only do applied problems demonstrate the usefulness of mathematical concepts, they provide context and meaning to the content; in particular, incorporating mathematical modeling "helps to bridge the gulf between reasoning in the mathematics class and reasoning about a situation in the real world." [4] Real-life applications also result in an incentive to retain the material. This general consensus is akin to the age-old student question, "Is this going to be on the test?" If students don't perceive mathematical concepts as useful for problem-solving or to be used in the working world, the knowledge is never really gained, having only been acquired for 'the test'; it is lost.

As a pure mathematician, Dr. Brooks acknowledges that she can be lax on requiring her students to answer applied problems with the appropriate units. Mathematics professors are often times most concerned with whether students are able to demonstrate an understanding of concepts as well as the ability to solve problems. Whether the final answer is right or wrong is less of an emphasis. On the other hand, Dr. McDonald, being an engineering professor, is very concerned with the final answer and the appropriate units on his problem sets. This was quickly discovered by Dr. Brooks on the second midterm in Statics, when Dr. McDonald deducted points on her exam for a simple arithmetic error. Neither instructor is "right or wrong" on their stance; however, what needs to be stressed is for students in both disciplines to effectively communicate their solutions and answers.

Answering an engineering problem with a mere numerical answer is not acceptable; the units often convey as much information as the value. In addition, each step of the solution needs to be explained so that anyone trying to follow the work later can easily interpret what was done. Not just anyone benefits from learning to do this; in the practice of engineering, problems are solved, reports are written, and the job notes are filed away. Months or years later, a question will inevitably arise. If the job notes cannot be deciphered, precious time, money, and reputation is lost. High-value engineers learn to document their work. As instructors, we elevate the value of graduates by teaching them to document and explain their work… using English! Go ahead, demand it. A student once asked a professor, "Why do I have to show all this work – don't you already know how to do it?" The response, "I do! And, I want to know that you do, too."

Students need to be groomed in appropriately phrasing answers to problems as early as introductory calculus and physics courses so that good habits form throughout their undergraduate experience. If students don't learn to explain their work while in school, they won't be able to do it in the workplace, and academia will have failed the profession. There is no better place to demonstrate (or for students to learn) the process of explaining the steps taken to solve a problem than in the development of a mathematical model for an engineering application. By having to explain what the math is doing and being used for, step by step, the student is also observing and learning the role it plays in arriving at the final equation. Later on, as a practicing engineer, there is an understanding that just because I didn't integrate r dθ today, calculus is still the means of obtaining πr^2.

As students progress into their engineering courses, they find themselves immersed in a world where math and science occur simultaneously. Typically, each week of an engineering course introduces at least one new concept with its accompanying vocabulary, constants, symbols, *model*, and resulting equations. As an engineering instructor, it is easy to begin believing that there just isn't enough time to cover all the required material. It is tempting to cut corners. Technology is bounding ahead and there is important 'new stuff' that engineers need to know. By racing through the traditional fundamental topics (the modeling and the math), time is carved out to introduce and develop proficiency in new leading-edge technology.

Don't take the bait! In our quest, it is embarrassing to not be able to distinguish between señor and señora.

Rather than efficiently deriving the model and equations step by step at the board or through a slide show, slow down and involve the students. Make the derivation an interactive process. Find ways to get students to engage the topic, the demonstration, and each other; get them out of their seats and doing something: move chess pieces, empty/fill containers, break spaghetti noodles. Find physical things to do that relate to each topic. By adding engagement into the lessons, they will be tricked into learning; it's unavoidable! There are many references on the benefits of engagement in learning, but Heather Wolpert-Gawron sums it up nicely, "It's become a part of our responsibility to not only teach the content, but to teach it in a way that stands a chance against the competition. And the only way to do it is to tackle our students' levels of engagement." [5] Don't avoid developing models and the mathematics; make it something the class wants to do, looks forward to, and expects.

While they are working things out, don't shy away from completely solving problems. By repeatedly working through the difficulties of problem development, students observe that most impossible tasks can be completed with an appropriate level of preparation, concentration, and determination. If needed, *let* them take it outside of class. In fact, when they start taking it outside on their own, you will know that they are experiencing a learning technique dubbed challenge-based instruction; [6] they can't lay it down. Not every student will arrive at this level, and those that do may not rise to this level with every topic; but, when they do, it's exciting. Students get there by facing the math head-on. They overcome the stigma that "it's hard" and realize that it just takes preparation and a solid foundation to move ahead.

Within both engineering and mathematics, critical thinking and self-assessment skills need to be honed rather than assumed that they are already developed. Peter C. Brown worded the importance of self-assessment nicely: "To become more competent, or even expert, we must learn to recognize competence when we see it in others, become more accurate judges of what we ourselves know and don't know, adopt learning strategies that get results, and find objective ways to track our progress." [7] When faced with a mathematically complicated problem, the question that students first ask themselves is "How? How do I solve this?" While this is an appropriate query, it should not be the only one. We need to insist on our students asking "What? What am I solving for? What type of a problem am I facing? What are the implications of the hypotheses? What is a reasonable outcome? What information do I need to solve this problem?" Just as important is "Why? Why would this method of solution be appropriate? Why am I getting these results? Why can't certain approaches be applied under these assumptions?" Pausing to consider such questions aids students in identifying the most efficient way to mathematically approach a problem rather than diving into the first approach that comes to mind. As Schoenfeld stated in regard to student self-regulation, "It's not only what you know, but how you use it (if at all) that matters." [8]

In addition, while solving problems, students need to pause mid-solution and evaluate their work. Some appropriate questions to reflect on include, "Is this solution heading in the right direction? Do these outcomes seem reasonable?" Too often, students become engrossed in a certain approach, and they are unable to recognize that their solution will not lead them to the correct answer. The inability to think critically is very damaging for our students. Within *The Power of Mindful Learning*, author Ellen J. Langer perfectly quantifies the importance of deep conceptual understanding of material and critical thinking:

> Closed packages of information are taken as facts. Facts are taken as absolute truths to be learned as is, to be memorized, leaving little reason to think about them. Without any reason to open up the package, there is little chance that the information will lead to any conceptual insights or even be rethought in a new context. We can think of such encapsulated information as overlearned. [9]

Within any STEM discipline, there is often an emphasis on problem-solving. The beauty of mathematics is that for a given equation there is usually only one correct answer; however, in engineering, there are often many modeling options with each resulting in a set of equations that provide a solution. The various solutions are not necessarily the same or equivalent; they are model dependent. When instructors demonstrate multiple ways to approach a problem, it provides students with a broader understanding of the material. One approach may "click" with one student but not with another. In addition, seeing that there is more than one approach and that different approaches can result in solutions that are sensitive to different aspects of a problem is an important lesson.

Mathematical models of fundamental physical phenomena are used to build models for more complex situations. The next section discusses using fundamental building blocks and some of the above interventions in the context of teaching soil consolidation to junior and senior level students.

Discussion of Soil Consolidation

Soil consolidation is a process induced when loads are applied to a saturated stratum of clay and was first recognized a hundred years ago by Karl Terzaghi. [12] Increasing the load increases stresses within the clay and 'consolidates', or reduces, the void space within the clay. The clay is saturated, meaning the void space is filled with water, and a reduction in the void volume requires that water in the voids must be discharged from the voids and flow to another location. Soil consolidation models utilize principles from mathematics, solid mechanics, and fluid mechanics. Students learn these principles in trigonometry, calculus, differential equations, physics, strength of materials, thermodynamics, and fluid mechanics.

Consolidation results in a change in volume that is expressed, near the surface, as settlement (often of the structure that caused the increased stress). Unexpected settlement can be detrimental to a project, not only to the physical facilities but also socially. As an example, consider the recent public outcry over the unanticipated differential settlement of the Millennium Tower completed in 2008 in San Francisco, California. [11] By 2018, the iconic skyscraper had settled seventeen inches. Introducing a topic by using an example in the news adds credibility and purpose to the learning, provides opportunities for interaction and engagement, and opens the door for additional discussions on topics such as professional ethics, sustainability, and societal needs. Students frequently return to class having read more of the news articles and now have an interest in learning about the "how" and "why" of the reported event. They want to figure it out. It makes the underlying math fun, meaningful, and memorable.

In basic soil mechanics courses, solutions for soil consolidation problems are taught. The standard solutions are developed in most introductory geotechnical engineering textbooks, such as Holtz and Kovacs, and students practice applying the solutions and using equations presented in the text to solve textbook problems. [10] The model and solution can be developed in class from fundamental principles, or the equations can be employed directly. Applying the equations may represent the use of mathematics, but this usually only involves a bit of algebra for most of the everyday problems, even in professional practice. However, when those special problems come along, the ones that don't quite fit the textbook model, the students that learned to do more than just work the equation, the students that learned how to derive a solution from basic principles, are able to identify the new twist in the problem and then adjust the model and obtain a more appropriate solution. This is where the math is in engineering!

The soil consolidation problem is a great means of demonstrating how to apply prior knowledge to a new situation. In a soil mechanics class, students already know how to set up a control volume (thermodynamics), the principles of fluid flow (fluid mechanics), and probably even about seepage in soil (a week earlier). They know about internal stress (solid mechanics) and that strain accompanies stress. Learning how to apply the knowledge of these principles to the solution of a new situation, such as soil consolidation, is an important skill – one that cannot be learned by simply applying the equations that someone else derived. Students need to practice the critical thinking required to derive the solutions. When students master these skills, they approach problem solving from a different point of view and in much greater detail throughout their life than the students that simply grind equations.

Conclusion

Where is the math in engineering? Well, it's still there, it just needs to be used. Instructors need to meet it head on. When used throughout the curriculum in coursework, students will continue using math in their future professional work. Providing students an engaging environment that fosters well-developed, continuous, practiced use is what is required. When involving math in the solution becomes the preferred path (rather than simplifying or adapting a problem to use an equation), then as professionals, they will reply to inquiring students that they "use it every day."

Indeed, mathematics is the foundation of the sciences, and engineering is no exception. For engineering students, fluency in mathematics is not only encouraged but required. However, simply gaining a strong knowledge of math is not enough for a student to become a successful engineer – an engineer that solves problems rather than one that simply calculates answers. Students need to be able to both apply and communicate mathematical concepts as they apply to engineering concepts. However, these skills are seemingly becoming a "lost art" among students and practicing engineers alike.

When it comes to applying mathematics within engineering coursework, both students *and* faculty need to take responsibility. It is the students' responsibility to set aside an appropriate amount of time to regularly work on course materials outside of the classroom, not take on too many outside responsibilities (too large of a course load or employment commitments), maintain healthy sleep habits, and the like. Without these basic human necessities, students (and working engineers) likely won't be able to focus on the more challenging tasks required by innovation. In an environment of deprivation, it's not long until the ability to create is lost – there is no time!

On the other hand, both mathematics and engineering instructors need to set aside time to aid students in applying math to engineering problems by being more conscientious about honing students' critical thinking and study skills. Engagement during class needs to be a priority; interaction between students (peer to peer) and with the instructor makes learning something to look forward to rather than drudgery. Citing examples from current events, introducing an enlightening history, and providing an overview using simple demonstrations makes content more relatable and interesting. Better yet, have students run the demonstrations themselves.

Student confidence is bolstered by practicing applied problems within mathematics courses. In applied courses, including engineering, instructors should strive to make connections with mathematical concepts to prior courses as well as foreshadow future courses. Prerequisites exist to introduce and develop knowledge required in subsequent coursework. During those subsequent courses, don't simply reference that knowledge – get students to actively apply it. Get off the yellow brick road, Toto, we're not in a math class anymore; the theorems *actually* have applications. Students figure out how to use math by (*gasp*!) applying math outside of math classes. ¡Adiós, amigos!

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