Why Is Teaching Problem Solving So Difficult?

(and how to make it easier)

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Abstract

This paper discusses the difficulties experienced by teachers and students of mathematical problem solving in engineering education. The particular sources for these difficulties are pointed out and suggestions for dealing with them are given. The primary sources of difficulty are: language and terminology, textual and written materials, and student attitudes and expectations. Included is a discussion of the understanding that it is not sufficient to teach engineering mathematics with the assumption that when that is well taught and learned, the ability to solve problems necessarily follows. Intentional teaching of problem solving per se is required.

Introduction:

Though there is increasing promotion of teaching problem solving by mathematics educators and various professional organizations, we still struggle with exactly what that means, with how to do it, with texts and materials which often actually hinder rather than help, and with students who are unprepared and/or reluctant to engage in problem solving. This paper discusses both overt and subtle reasons why teaching problem solving is difficult in pre-engineering education and in engineering education classrooms. It also suggests techniques and approaches teachers can use to ease the difficulties teachers and students experience in attempting to solve mathematics based problems. Otung suggests that it may help to de-emphasize the mathematics in the traditional initial stages of engineering education in favor of a focus on engineering problems.[11] The concern apparently is that an initial difficult experience with problem solving may preclude adequate future work in problem solving.

Several aspects of the origins of problem solving difficulty students face will be examined. These include: language effects on problem solving, textual materials issues, and student attitudes toward, and understanding of, problem solving, and with each are suggestions for teaching strategies that will help deal with the difficulties. The last section discusses the intentional teaching of problem solving and useful strategies to use for that.

Language:

First, problem solving language, both written and spoken, is a critical concern. The language used by teachers, students, and texts can be helpful or cloud the whole problem solving process. There are words and phrases which, although common in our everyday conversation, promote misconceptions about problem solving.. These include such phrases as: 'answer a problem,' 'work a problem,' 'do a problem,' and 'word problem.' Problems are solved, not done or answered! In addition the word 'problem' is used to describe exercises and situations which are not problems. They may require complex processes or are difficult for some other reason. They are then called 'problems' though they are actually exercises that will yield to the application of some algorithm. To use the word 'answer' in connection with problem solving is to suggest one number or word as a solution and that there is one algorithm for obtaining that solution.

An example comes from Kreyszig's text, Advanced Engineering Mathematics.

At the end of section 4.1, there are "Problems for Section 4.1." Twelve 'problems' (which are actually only exercises) are listed after the statement, "Apply the power series method to the following differential equations." 1. y' = 2y 2. y' + y = 0, etc. and these **exercises** statements are followed by, "(More problems of this type are included at the end of the next section.)"[7] At the end of the text itself is a section labeled "Answers to Problems" in which the listings are in the format: 1. $y = e^{-5x}$,
3. y = , etc.

This example is illustrative of the format and kinds of textual and problem solving language which is all too common in engineering problem solving education. It only enhances the already poor concepts and understanding students have about mathematical problem solving. The word 'solution' also presents difficulties. When used as a synonym for 'answer', the subtle connotation is that, as in computation, there is just one number, which is the desired result. The phrases are used as a result of these misnomers. A better understanding of the idea of a solution includes the sense that if the result does not meet the requirement of the problem, it is not a solution. The phrase 'correct solution' is a redundancy. Similarly, the phrase 'complete solution' is also redundant. If it is a solution, it is necessarily complete. If not, it is not a solution.

In contrast to what students experience in computation, a solution is typically not just one number. True problem solving results in solutions in varieties of forms including diagrams, series of numbers, patterns, general formulas, explanations, and even algorithms. The phrase 'multiple solutions' presents a difficulty of a different kind. Several problems have multiple sets of numbers or explanations, each of which meets the requirements of the problem (i.e., solves the problem). There is still a question about how best to describe this situation. The solution may be thought of as the set of all these 'partial' solutions or, perhaps, a satisfactory understanding is had from the phrase 'multiple' solutions.

Caution should be taken not to confuse 'solution' with 'strategy'. Cai, in a helpful article on teaching problem solving, discussed multiple strategies for solving the same problem, but called them 'multiple solutions'.[2] The underlying issue may be a lack of understanding of the concept of problem solution and problem solving. In working with technology in engineering education, Mioduser suggested the need for defining an appropriate conceptual framework supportive of requisite knowledge, skills and cognitive models for problem solving.[9] Thus it will be important to endeavor to make language supportive of concepts that are critical to problem solving success and to make sure that the language used reflects the underlying concepts involved in problem solving in engineering.

Suggestions for dealing with the difficulties posed by language usage include the following:

- Use correct terminology and urge students to do the same by pointing out to students the difficulties created by poor language concerning problem solving.
- Help students to understand the difference between problems and exercises.
- Teach intentionally and directly what constitutes a problem and a solution.
- Help students learn that there are often different strategy combinations that may be used to reach solution.

Texts:

Secondly, textual and written materials concerned with problem solving are examined. Several aspects of these materials make teaching problem solving difficult. One is that problem solving is often left as an afterthought. It is left to the end of the page of exercises or to the end of the chapter where it is housed with the 'more challenging' tasks. The implication is that once the symbolic manipulation tasks are learned and mastered through work on the exercises, then problem solving can be done. Furthermore, it is often assumed that, having learned the symbolic manipulation techniques, students will be able automatically to solve the problems where those symbolic manipulation techniques can be applied. Typically the only problems presented are 'word' problems and many of them are contrived. Realism and practicality have given way to esoteric statements designed so that particular symbolic manipulation techniques can be directly applied. De La Barra, in an attempt to cope with freshman engineering students who lack necessary cognitive skills, propose a 'new teaching scheme' which involves the "systematic use of routine steps that constitute the whole problem solving process."[4] This approach appears to make problem solving very algorithmical and may be counterproductive in the long run. It may also be that some are attempting to make problem solving into formats that appear very much like a series of steps to learn because of the necessity of putting the textual discussion in easily readable formats. If that is the case then what is presented is not actual problem solving at all. It is only applications of algorithms for certain 'types' of problems. For example: If it is a 'rate problem,' then students learn that they should make a grid chart and fill in each of the boxes. They understand that if this grid is filled out carefully, one of the boxes will contain the 'solution.' In effect, then, the activity is really only an exercise to which an algorithm is applied and not problem solving at all.

Texts that purport to have the solutions in the 'answer book' or 'answers' section at the back of the book cause a further set of problems. First the list of 'solutions' often contains only the results of exercises, not solutions to problems. Solutions aren't often one number items; they are much lengthier explanations and descriptions. To have cryptic 'solutions' in the back of the book suggests that what is really referenced are exercises, not problems.

These back-of-the-book presentations also suggest that what is listed is the only and/or right way to reach solution. Because this resembles the back-of-the-book work for computation exercises, students tend to use them in the same way as they use computation answers. If the students have done actual problem solving, they will often have solutions that look very different from what is in the back of the book. This obviously creates unnecessary frustration and uncertainty for problem solvers.

Suggestions for dealing with the difficulties teachers of problem solving face because of textual materials include:

- Start sections and lessons with meaningful problem solving so that exercises are done to support problem solving rather than the reverse.
- Use texts which do not have 'answers' to problem solving listed.
- Make clear distinctions for students between exercises and problems.
- Do not teach problem solving as learning algorithms for particular 'types of problems.'
- Teach problem solving intentionally using problems on the topic but problems which are solved using strategies different from those used for other problems.
- Make it clear to students that mastering symbolic manipulation alone will not make problem solving happen.
- Make it clear to students that symbolic manipulation skills are tools to be used along with others in problem solving.
- Give problem solving examples independent of 'type' and practice demonstrating problem solving using different and multiple strategies.
- Avoid giving the impression that students are to memorize or remember the steps in the solving strategy for one problem on the assumption that another problem can be solved using the same series of steps.
- Help students to understand that, when the need for a process or complex computation is required during problem solving, students may use exercises to practice and develop a problem solving step so that, on returning to the problem solving process, they will be able to proceed effectively.

Student Attitudes and Expectations:

Thirdly, teachers experience difficulties with student attitudes toward, and misconceptions about, problem solving. Students are conditioned by an overwhelming emphasis on computations and related algorithms, to believe that these constitute all of mathematics. Students then naturally try to solve problems by the same methods and techniques they learned for computation. That is, they memorize or try to find an algorithm that will work. When this proves impractical or difficult, students become frustrated and angry or withdraw from problem solving because they judge it to be too difficult or impossible. They may also believe that it is not as important as symbol manipulation because it has received so little emphasis in their science and math classes. It is also easy for students to feel that problem solving is theoretical and not actually useful for real physical situations. They have had little experience in creating problems or in seeing and understanding the problems in real practical settings. This means that they may believe that engineering problems are contrived or, at best, found only in textbooks. Otung suggests that an over-emphasis on mathematics for beginning engineering students causes them to feel that there is an 'unfriendly gatekeeper' at the entrance to engineering so that a negative attitude toward problem solving develops when students begin engineering studies.[11]

Some of the subtle misunderstandings and misconceptions students have about problem solving are:

- Unwillingness to bring into problem solving anything from outside the problem statement,
- A one-trial then quit mentality,
- The assumption that every problem statement of a solvable problem contains key words or clues to tell the problem solver what to do,
- The idea that all mathematics is cumulative and hierarchical,
- Every problem has one solution containing no parts.
- From the 'trial and error' phrase, the idea that if a trial didn't yield the solution, then something wrong was done.
- That mathematical activities are done quickly, if the solver is competent.
- That most students do not have the necessary skills for problem solving.
- That there is only one strategy or method for solving a particular problem.

Suggestions for dealing with student attitudes and misconceptions about problem solving are:

- Pose interesting and real-life problems which students do not have to struggle to understand.
- Demonstrate to students that they have the 8 problem solving **skills**, the abilities to:
- Classify, Deduce, Estimate, Generate patterns, Hypothesize, Translate,[6] Try and Modify, and Verify. (Ito)
- Give problem solving examples illustrating the application of these skills, and give practice that results in students sharpening these skills.

- Demonstrate the necessity of bringing into the problem solving strategies, any information from any source that may contribute to the solution.
- Demonstrate that since multiple strategies are available, problem solving is not necessarily hampered because some particular mathematical tool is unavailable.
- Show that trials which do not lead to solution usually provide useful information to guide re-trials, that errors have not been committed, and that trial information should not be destroyed until after solutions are reached.
- Remind students that reaching solutions often takes time and that experimentation is to be expected.
- Remind students that there are no algorithms for true problems so they should not waste effort in trying to remember 'how we did this one the last time'.
- Teach students that careful reading and comprehending the problem statement or situation are necessary and the search for 'key words' is likely to be counterproductive.
- Give students practice with 'multiple' or 'conditional' solution problems.

Methodologies:

In order to cope with some of these aspects of poor entry skills and student attitudes, De La Barra recommends stratifying course content in an attempt to enable students to deal with easier material initially and then proceed to more complex.[3] This raises the question of whether it is possible to stratify the problem solving aspects of the beginning courses in engineering.

General methodologies for teaching problem solving should include intentional teaching of problem solving per se. Students need to understand that they each already possess the necessary skills. (Individual skills may need to be refined and practiced to become more readily useful and effective.) Students should also be taught that to solve problems, the skills are applied in some sequence to form a strategy or series of strategies which will lead to solution. Practice then should be given in problem solving using a variety of types of problems and problems which can be solved by a variety of strategies. Willamowski and colleagues indicate that a problem solver must be able to intervene in the problem solving process and make choices along the way and that these choices will affect the various aspects of the process. An example is that the solver might change parameter values experimentally in order to understand the problem more fully or to find strategy selection clues. Included in the activities and instruction about problem solving, then, should be reminders and examples of the following:

- Multiple strategies are possible and encouraged.
- The solution may take a variety of forms.
- Problem solving is time consuming.
- Trials and re-starts are necessary and may provide additional information useful in the problem solving process.

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