

Work in Progress: A Structural Change in Calculus Sequences

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Introduction

As our society has become technology reliant, the need for a STEM trained populace grows [1],[2]. Despite this growing need, the US is behind other nations in math and science academic performance and in the percentage of STEM majors [1]. Noonan [3] demonstrated that STEM career options are expanding. However, nearly half of those majoring in a STEM field leave the field [4]. Research has found mixed results regarding retention when comparing STEM majors to non – STEM majors [5], [6].

The gender gap in STEM persistence is substantial [7]. Overall, 65% of men persist in STEM majors while only 48% of women persist. This gap is even larger in computer science with a 64% to 43% persistence rate. Persistence rates differ along racial lines as well. The six-year graduation rate for students from underrepresented groups is between 22% and 29% while white students have a six-year completion rate of 43% [8].

Identifying roadblocks to STEM success and persistence is vital for the future of STEM. Crafting mitigation plans aimed at student success should be research based and implemented to welcome and benefit all students. Researchers have worked to identify predictors of STEM persistence, both before matriculation and after. A student's level of academic success before matriculation is a strong predictor of STEM persistence. These predictors include standardized test scores and taking calculus in high school [9], [10].

Research has found that, after matriculation, a student's likelihood to complete an undergraduate degree was linked to a student's level of academic and social integration. Tinto [11] defines academic integration by a student's academic performance and their perception of their own academic experience. Therefore, it follows that academic integration and experience are influenced by course contents, offerings, and pedagogy. Academic integration can also be linked to academic success [12]. This creates a cyclical relationship among these factors. Therefore, improving a student's academic integration will have an impact on their academic success, positively impacting STEM success and persistence rates.

Poor mathematics performance is a leading factor for leaving a STEM major or a degree program [9]. The calculus sequence has long been identified as a roadblock to academic success for STEM majors. More specifically, the first calculus course stands out as a predictor for persistence and retention in STEM majors [4-6], [8-10], [12-15]. Ohland [13, pp. 253] found that failing or withdrawing from calculus had a bigger impact on degree progression than failing other courses. For engineering students, the "grade a student receives in their first math course is a singularly important factor in predicting retention and graduation with the level of the course being somewhat less important" [12].

Research has identified program attributes linked to persistence in STEM majors and academic success. Ohland [13] states that students need to start at a point in the math sequence that fits their preparedness. Pearson et al. [14] conducted a systematic review of STEM degree support models and identified key components of success. They include clear expectations and student

support services that enable students to meet those expectations, adequate financial support, offering experiences that combine social and academic components, and increasing a sense of belonging and STEM identity. Bressoud et al. [15] assert seven characteristics of successful calculus programs. Characteristics include making data driven decisions, effective placement procedures, student centered pedagogy, and support services that foster academic and social integration.

Handelsman et al. [16] issued a call to “*fix the classrooms.*” They assert that reform movements have been slow to show progress and that change has been one-sided and focused on fixing the students. Their call to action asks administrators and instructors to focus on fixing classroom experiences. Inclusive practices, active learning, and early research opportunities are key components of their action plan.

As part of the effort to fix the classrooms, a Split Calculus Initiative is a multi-pronged plan launched in Fall 2022 at Wentworth Institute of Technology (WIT). Wentworth’s Split Calculus Initiative sequence includes many of these previously discussed attributes. The initiative starts with a math placement program that aims to place students at their most advantageous mathematical starting point. A free summer mathematical ‘booster’ experience builds both community and mathematics preparedness. Each class section uses embedded tutors, faculty lead facilitated study groups, and first year advisors to build a cohesive support network for student success. In the spirit of a student ready institution [17], the Split Calculus Initiative is a student-centered approach to course offering that provides the needed academic structure with additional onramps for mathematical success. In this paper, we will discuss the structural changes made in offering the calculus sequence at WIT and the other new initiatives aimed at student success.

Data Prior to Structural Changes in the Calculus Sequence

Previously, Calculus 1 and II at WIT were delivered as traditional semester long courses. MATH1750 is the Calculus 1 course for engineering students, and MATH1775 is the Calculus 1 course for computer science and applied mathematics students. Since the Fall of 2018, 2355 students have enrolled in MATH1750 and 728 students have enrolled in MATH1775. Note here that when a student retakes a class they will be counted twice. Of those 3083 students enrolled in Calculus 1, 745 students either withdrew or failed (574 of the 2355 students in MATH1750, and 171 of the 728 students in MATH1775), and thus did not complete their class (see Table 1). At Wentworth, receiving a letter grade of a D allows a student to continue to the next course in the sequence, therefore, failure and withdrawal (FW) rates are being reported as opposed to the DFW rate.

Table 1: Fall 2018-Fall 2022 Failure and Withdrawal Rates for Calculus I

	MATH 1750	MATH 1775	Total
Pass	1781 (75.6%)	557 (76.5%)	2338 (75.8%)
Fail or Withdraw	574 (24.4%)	171 (23.5%)	745 (24.2%)
Total	2355	728	3083

A Pearson’s Chi-Squared test for independence and Fisher’s exact test on this data (p-values 0.6615 and 0.6558 respectively), resulted in not rejecting the null hypothesis that the class a student takes (MATH 1750 or MATH1775) is independent from the student’s outcome (Pass or

WF). For this reason, data from MATH 1750 and MATH 1775 will be treated as one data set in subsequent analyses.

It is important to explore the impact of any structural changes on students from underrepresented groups. Therefore, the 2018-2022 Failure and Withdrawal data was analyzed for differences in ethnicity and gender (See Table 2 and Table 3).

Table 2: Fall 2018-Fall 2022 Failure and Withdrawal Rates for Calculus I by Ethnicity

	White	Black/ African American	Asian	Hispanic	Other	Total
Pass	1385 (79.1%)	165 (56.9%)	272 (80.0%)	251 (67.8%)	265 (79.8%)	2338 (75.8%)
Fail or Withdraw	366 (20.9%)	125 (43.1%)	68 (20.0%)	119 (32.2%)	67 (20.2%)	745 (24.2%)
Total	1751	290	340	370	332	3083

A Pearson’s Chi-Squared test for independence resulted in rejecting the null hypothesis that ethnicity (White, Black/African American, Asian, Hispanic, or Other) is independent of the student’s outcome (Pass or WF) ($p\text{-value} < 2.2(10)^{-16}$). In particular, the FW rate for the Black / African American students is significantly higher than the other groups combined ($p\text{-value}$ of $4.387(10)^{-15}$).

Table 3: Fall 2018-Fall 2022 Failure and Withdrawal Rates for Calculus I by Gender

	Female	Male	Total
Pass	406 (79.8%)	1932 (75.1%)	2338 (75.8%)
Fail or Withdraw	103 (20.2%)	642 (24.9%)	745 (24.2%)
Total	509	2574	3083

Similarly, using either Pearson’s Chi-Squared test for independence or Fisher’s exact test ($p\text{-value}$ 0.027 or 0.023 respectively) resulted in rejecting the null hypothesis that gender is independent of student’s outcome (Pass or FW). Analysis supports that gender and the student outcome are not independent. Specifically, female students have a significantly higher passing rate. Predicting student success based on ethnicity, gender, and all the interactions between those variables using a logistic regression model is significantly better than a null model ($p\text{-value} < 9.83(10)^{-16}$ using a drop in deviance test). However, in this model no individual factor was significant in predicting student success (all $p\text{-values} > 0.05$).

These findings reflect previously discussed research regarding persistence for students from underrepresented groups [8]. One difference in this data is the higher success rates of female students in comparison to male students.

Not completing Calculus I, creates a substantial challenge in degree progression. Five years of data demonstrated that more than 24% of Calculus I students needed to repeat the course. This clearly illustrates the ‘calculus bottleneck.’ Calculus is a prerequisite to the physics sequence, upper-level mathematics courses, and many engineering courses. Needing to repeat calculus prolongs degree completion, negatively impacts a student's perception of their academic success,

and feeds the persistence issue in STEM majors. This is the exact scenario Middleton, et al. [12] found as the principal factor for engineering students' persistence to degree completion.

Structural Changes to the Calculus Sequence at Wentworth Institute of Technology

The discussed changes here are not the first attempts at restructuring the calculus sequence. For historical context, in the summers from 2013 to 2017, an NSF-sponsored Summer Bridge Program (SBP) was offered to local incoming students deemed at risk of failing their first mathematics course. Each cohort had between 25 and 30 students. Students who participated in this summer bridge program performed as well as their peers in Calculus 1 and 2. (See Table 4).

In particular, SBP students had a higher average Calculus I grade than the rest of the students taking this course who did not participate in SBP.

Table 4: Fall 2013-2017 Calculus 1 Grade Point Average for SBP and Non-SBP students

Semester	SBP GPA Calculus 1	Non-SBP GPA Calculus 1
Fall 2013	3.1	2.8
Fall 2014	3.3	2.7
Fall 2015	2.8	2.6
Fall 2016	3.1	2.5
Fall 2017	2.4	2.3

Similarly, in the Fall 2011 to 2014 at the mid-semester point of Calculus I, students at risk of failing were offered the opportunity to restart Calculus I in a special “reboot” section of the course. This reboot version spanned the remainder of the in-person semester and continued online during the semester break. Successfully completing the reboot version of Calculus I enabled students to proceed to Calculus II the following semester. This opportunity provided a way through the calculus bottleneck and provided a pathway for progress toward degree completion.

In addition, multi-tiered academic support resources were offered to all calculus students. The first tier was standard faculty office hours, the next tier was peer tutoring through the Center for Academic Excellence (CAE). Additional study groups (Facilitated Study Group or FSG) were faculty led weekly with the additional support of peer tutors. Furthermore, Learning Labs for Calculus 1 and 2 were offered. Learning Labs was a program initially financially sponsored by the NSF, then funded by the university. These learning labs are 2-hour long review sessions, run by upper class student leaders for students in these calculus classes. The Learning Labs were an additional resource which supplements the FSGs, CAE peer tutoring, and faculty office hours.

Split Calculus Sequence

Despite the changes described, WF rates did not improve to desired levels (See Table 1). Faculty explored multiple options. After researching calculus initiatives at other institutions, one institution’s structural changes were an excellent model to emulate. Binghamton University SUNY implemented several changes in its calculus sequence offerings in fall 2014. In the years

following, persistence to degree completion increased. They also saw extraordinary improvements over multiple markers for female and students from underrepresented groups [18].

The Split Calculus Sequence was created as a multi-pronged action plan that includes a new math placement process, a new Introduction to Calculus course, breaking the full semester courses into two seven-week offerings, and building a robust summer program to both prepare students for the calculus sequence and provide opportunities to complete needed prerequisites for second year course work. This structure addresses barriers that a traditional calculus sequence may create impeding progression toward degree completion. The overarching goal is to create a flexible system of curricular offerings and support services to best facilitate student success.

The math placement process (MP) follows the ideas of Ohland et al. [13] who found that students' first mathematics course should be matched to their preparedness. All incoming students are required to take a placement exam. This is a thirty-problem online exam meant to discern which of four course options matches a student's skill set. Those options include College Mathematics, Precalculus, Introduction to Calculus, and Calculus I (See Figure 1).

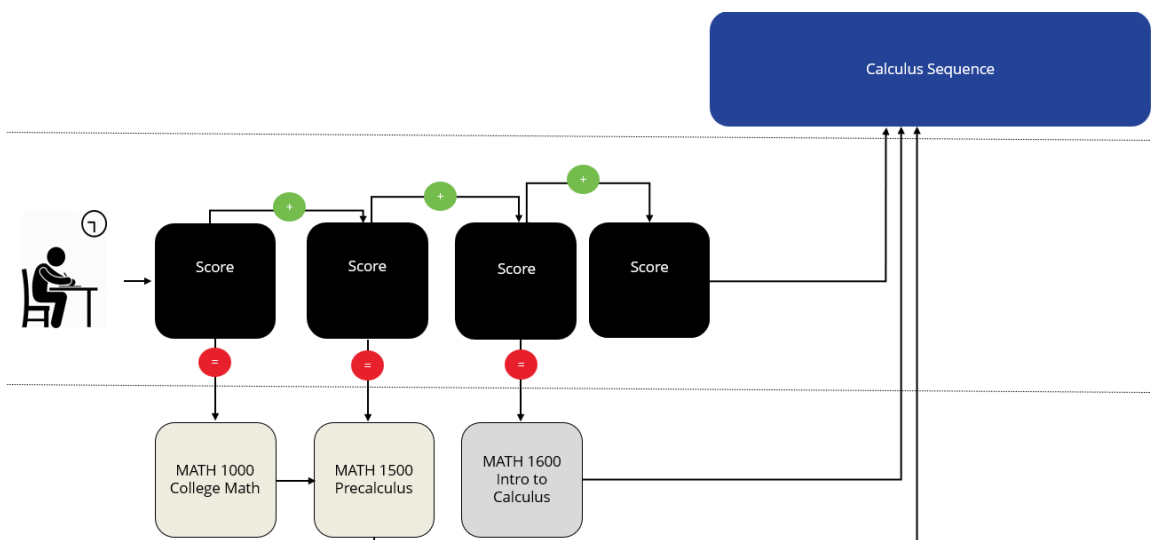


Figure 1: Model of the Math Placement process.

Adjustments are made for transfer, IB, and AP credits. As part of the placement process, students who place below their desired mathematics level are offered a free, non-credit summer course to help build and refine skills for the next course. Students can choose to take the course identified by their MP or take the summer experience and proceed to the next mathematics course upon matriculation. To best meet students' needs, these summer experiences are offered in multiple modalities (synchronous, asynchronous, or on campus).

The next aspect gives the Split Calculus Sequence its name. The traditional full semester length calculus courses have been split into two seven-week courses. Calculus I is comprised of Calculus IA and Calculus IB. Calculus II is comprised of Calculus IIA and Calculus IIB (see Figure 2).

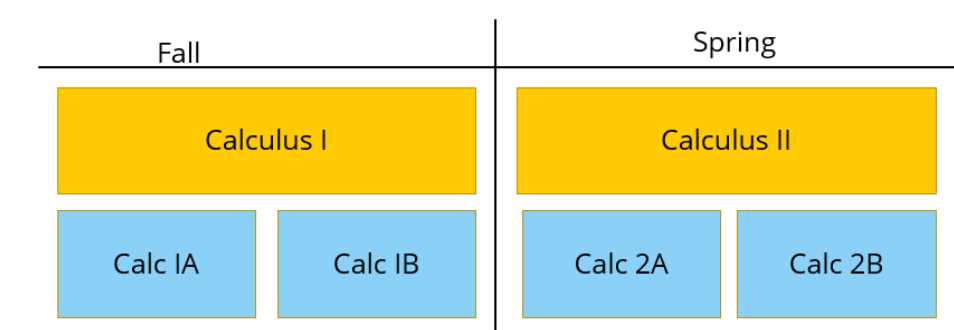


Figure 2: Initial Split offering of Calculus I and Calculus II.

The original four credit courses have been split into two, two-credit courses. The learning goals and course outcomes remain the same. Breaking the traditional semester into 7-week pieces has several potential benefits. It creates a rapid recovery option where unsuccessful students are not as far behind; this is especially valuable to students in tightly sequenced majors. If a F/W is earned in a new 2-credit 7-week course, a student can catch up in 7 weeks in the summer (See Figure 3).

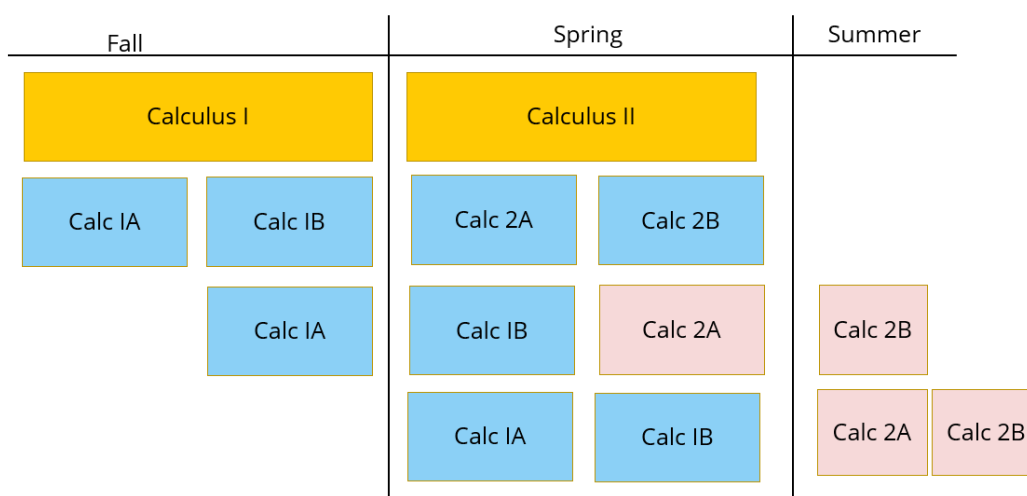


Figure 3: Split calculus sequence with multiple restart points.

Given that a failure at midterm can be a strong predictor of course failure, this structure enables an easier path to completing prerequisites than the traditional structure. Additionally, unsuccessful students can avoid wasting the remaining half of their current calculus course. Furthermore, the potential negative impact on GPA, and thus on financial aid, is mitigated by lowering the credit count for each class.

As the new structure continues to be rolled out, eventually all four seven-week courses will run simultaneously during each seven-week session. This will maximize curricular flexibility for those students who struggle at any point in the calculus sequence.

In addition to splitting Calculus I, a new course was added to the calculus sequence. Titled ‘Introduction to Calculus,’ this course is offered in the 2 credit, 7-week format. Acting as a primer for studying calculus, this course reviews algebraic, trigonometric, and polynomial functions while introducing the concept of a limit. In the past two years, this has been offered as a free summer course for first year students. Moving forward, this will become an integral part of the mathematics sequence (See Figure 4).

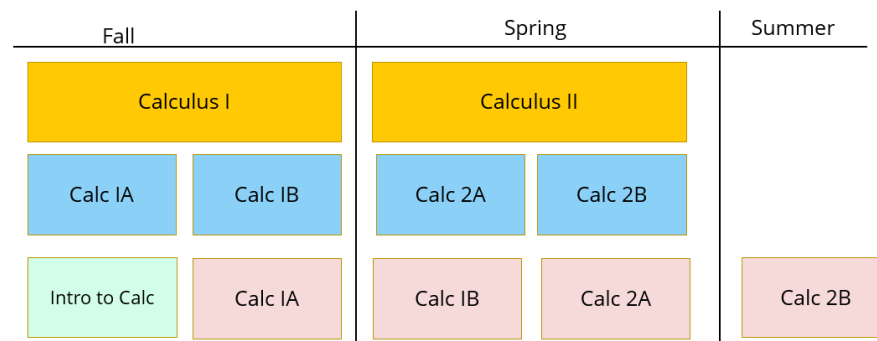


Figure 4: Split calculus sequence with Introduction to Calculus.

We would like to note that creating this new sequence creates some scheduling challenges for the registrar’s office. For instance, with multiple sections of Calc 1A in different time slots it is difficult for all students to maintain their exact schedules if they do not succeed in the 7-week course. We use bookend time slots of 8:00am and 5:00pm to help enable the rescheduling process. Additionally, we are working with a new registrar on creative ways to solve this issue moving forward.

Findings Fall 2022

In Fall 2022, there were 294 students enrolled in either Calculus 1A or Calculus 1B. As with the previous traditional sequence, students who move on from Calculus 1A or Calculus 1B will be counted twice, once for each section. Of the 294 enrolled, there were 64 students who either withdrew or failed (45 of the 178 students in Calculus 1A, and 19 of the 116 students in Calculus 1B). This represents a decrease in the FW rate from approximately 24.2 percent to approximately 21.8 percent (see Table 4). Note again students at Wentworth Institute of Technology can continue to progress with an earned letter grade of a D. Therefore, the FW rate is used.

Table 5: Fall 2022 Withdrawal and Failure Rates for Split Calculus 1A (1776) and 1B (1777)

	Calculus 1A	Calculus 1B	Total
Pass	133 (74.7%)	97 (83.6%)	230 (78.2%)
Withdraw or Fail	45 (25.3%)	19 (16.4%)	64 (21.8%)
Total	178	116	294

A Pearson’s Chi-Squared test for independence and Fisher’s exact test on this data (p-values 0.0963 and 0.0830 respectively), resulted in not rejecting the null hypothesis that the class a student takes (Calculus 1A or Calculus 1B) is independent from the student’s outcome (Pass or WF). For this reason, data from Calculus 1A and Calculus 1B will be treated as one data set in subsequent analyses. The p-values were low considering the small amount of data. We hope to re-examine this question as we have further semesters of data to evaluate. We anticipate that the trend shown in this preliminary data, where there is a lower FW rate for Calculus 1B, will become significant with more data.

Table 6: Fall 2022 Withdrawal and Failure Rates for Split Calculus by Ethnicity

	White	Black/ African American	Asian	Hispanic	Other	Total
Pass	102 (76.7%)	27 (77.1%)	42 (73.7%)	31 (93.9%)	28 (77.8%)	230 (78.2%)
Withdraw or Fail	31 (23.3%)	8 (22.9%)	15 (26.3%)	2 (6.1%)	8 (22.2%)	64 (21.8%)
Total	133	35	57	33	36	294

Table 7: Fall 2022 Withdrawal and Failure Rates for Split Calculus by Gender

	Female	Male	Total
Pass	54 (80.6%)	176 (77.5%)	230 (78.2%)
Withdraw or Fail	13 (19.4%)	51 (22.5%)	64 (21.8%)
Total	67	227	294

With this preliminary data using Pearson’s Chi-Squared test, we cannot reject either of the null hypotheses that ethnicity is independent from student outcome, and that gender is independent from student outcome (p-values 0.224 and 0.715 respectively) for students taking our new Calculus 1A and Calculus 1B courses. We also expect that these variables will show to be dependent as more data is analyzed.

It can be seen from the data (See Tables, 8, 9, and 10) that the split Calculus sequence has a lower overall withdrawal and failure rate compared to the traditional sequence, with a net difference of 2.4%. However, in a logistic model predicting student success (Pass vs Fail/Withdraw) based on sequence (Traditional vs Split), this drop in the FW rate is not statistically significant (p-value .396).

Table 8: Overall Comparison of Withdrawal and Failure Rates from Traditional to Split Sequence.

	FW Rate
Traditional Sequence	24.2
Split Sequence	21.8
Net Difference	-2.4

Table 9: Overall Comparison of Withdrawal and Failure Rates from Traditional to Split Sequence by Gender.

	Female	Male
Traditional Sequence	20.2	24.9
Split Sequence	19.4	22.5
Net Difference	-0.8	-2.4

Table 10: Overall Comparison of Withdrawal and Failure Rates from Traditional to Split Sequence by Ethnicity.

	White	Black/ African American	Asian	Hispanic	Other
Traditional Sequence	20.9	43.1	20.0	32.2	20.2
Split Sequence	23.3	22.9	26.3	6.1	22.2
Net Difference	+2.4	-20.2	+6.3	-26.1	+2.2

The comparison by gender indicates that the split sequence is more effective in reducing withdrawal rates for both females and males. When gender is added to the logistic regression model which predicted student success using sequence, gender is a significant predictor of student success (p-value 0.0207). The comparison by ethnicity reveals that the impact of the split sequence on withdrawal rates varies by ethnicity, with the largest decrease seen in the Hispanic group. However, in a logistic model predicting student success using sequence and ethnicity, none of the levels for ethnicity are significant predictors of success. Overall, the results suggest that the split sequence is an effective method in reducing withdrawal rates, especially for males, and that the effectiveness may vary by ethnicity. Further investigation is needed to understand the significance of these differences.

Going forward we will connect the results of our math placement with the results of the split calculus sequence. However, in Fall of 2022 our math placement exam was not yet binding, and so students were able to choose their own math class regardless of their placement score.

Conclusion

The STEM field needs to be proactive in the creation and preservation of academic pipelines. Roadblocks and hurdles need to be removed while maintaining desired learning goals. To this end, restructuring the calculus sequence aims at removing roadblocks while providing opportunities for success. By coupling a new math placement process with a newly restructured calculus sequence, the goal is to have a student start the sequence in a place that best suits their skill set.

Initial data from this restructuring suggests this restructuring shows promise. The failure and withdrawal rates dropped overall, and key demographics showed large gains. Drops in the percentage of Black/African American and Hispanic students who failed or withdrew is encouraging. Though many of the measures lacked predicting capabilities, more data is needed. While it is too early to measure year-to-year retention rates, that indicator will be a critical measure of success.

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