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Work-in-Progress: Mental Images in Studying Electromagnetism

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Abstract

The course on electromagnetism is a foundational course in undergraduate electrical engineering curricula. The course is challenging for instructors and extremely difficult for students.

Pedagogy in electromagnetism has been an active subject in the research literature. We can find many useful ideas about what and how to teach. Instead of the what and the how, our focus is on the internal mechanism of learning. Two basic questions come under the focus: a) What happens to a mind when it is getting to know a concept? and b) In what way can the grasp of a concept be observed? These questions are clearly important. The answers could have significant impact on our choice of teaching methods and materials.

The main point of the article is to make a case that mental images are a critical element in concept learning. We argue that when a concept is forming, certain mental images develop and evolve. A mental image may come into existence from none before; or it may result from modification and combination of existing images. It is possible that an old image becomes an obstacle and must be replaced by the new. In any case, development of mental images is essential for concept learning. We also think that internal imagery of a learner can be observed. It can be revealed through writing and drawing. Non-verbal outputs from the mind are especially useful to gauge the level of understanding.

We use three sources of information to make the case. First, research in cognitive psychology has long established that mental imagery is essential to all human thinking, particularly problem solving. Second, mental imagery and imagination in non-verbal form had been widely cited by scientists themselves. In the article, we cite direct statements made by physicists. Third, we present two cases of direct observation. Both strongly confirm the theory developed in the field of cognition science.

We expect that the emphasis on developing mental images in the fields course would lead to fruitful development of teaching methods and materials.

The paper uses a specific example to illustrate the idea.

Keywords electromagnetism, imagery-based pedagogy

1 Introduction

Almost all undergraduate electrical engineering programs have a course on electromagnetics. Often called the fields course, it is where students develop basic understanding of the classical theory in electromagnetism. The course is known to be very challenging for instructors and difficult for students. This reputation has negative consequences. With few exceptions, the subject of electromagnetics has seen a declining student interest and enrollment. Many electrical engineering programs have reduced credit hours for the course. The trend is not only happening in the U.S., but also worldwide, as shown by the reports from Finland[1], and France[2].

Students' grasp of the concepts encoded in Maxwell's equations is the main learning objective of the fields course. But, instead of learning the concepts, many students focus their efforts on finding formulas and procedural repetitions. The gap between the learning objective and the learning outcome is quite large and concerning. It is a main cause for dissatisfaction with the course. The question is how to move students away from regurgitating formulaic operations to concept learning. Many papers in engineering education literature have been written to address this issue. For instance, Leppavirta, Kettunen, and Sihvola[1] proposed the idea of integrating complex problem exercises into the process of problem solving. Notaros[3] proposed using concept questions, and integrating them with computational tools. The goal is to draw attention to concepts away from formulaic learning. Anderson and Mina[4] suggested a two-step teaching process. Students at first learn to describe physical concepts in words, and then in mathematics.

Our end goal is the same. But we first approach the problem with a different perspective—the perspective of students. We want to understand why the gap problem exists. We try to analyze the mental process of a student as a response to external stimuli. The premise is that with deeper understanding of the mental processes of our students, we may be in a better position to find ways to narrow the gap.

In Section 2 we describe the fields course and the typical background of our students. In Section 3, we raise the question: What happens when a student is getting a new concept? We argue that mental images play an essential role in the process while a concept is taking hold. The section is concluded with our proposal of imagery-based instruction. In Section 4, We contrast the process of formulaic learning with that of concept learning. We give an illustrative example to show what an image-based instruction may be like. In Section 5 we explain the difference between two entities: the graphical tools one uses and the mental imagery one possesses. Section 6 concludes the paper.

2 The Fields Course and The Students

This paper is a first step of our work-in-progress. The work aims to find better ways to teach the fields course. Here we give a sketch of the course we teach.

First, the students. To enroll in the fields course, students are required to have taken college physics, a calculus sequence, and basic circuit analysis. They have formed their concepts about charge, current, voltage, electric energy and power, Ohm's law, etc. All students have basic knowledge of vectors. Most of them are reasonably familiar with vector algebra, but not so much with vector calculus. Few students can say that surface integral or line integral in vector fields are tools in their mathematical toolbox.

Differential and integral vector calculus, particular operations such as divergence, curl, and gradient, are essential to the study of electromagnetics. By and large they are new to our students. So, they have to learn these tools in the course.

The course we teach expects two main learning outcomes. First, students should gain a fundamental knowledge of the classical electromagnetic theory; that is, Maxwell's equations. Second, they should be able to use the knowledge in some applications.

Our course is a two-quarter sequence. We have 20 weeks to achieve the learning outcomes. Approximately, students spend 10 weeks in developing and understanding Maxwell's equations. Other 10 weeks are used for application of the theory. To develop the full theory, students first study the mathematical tools. Then they go through a sequence the topics: eletrostatics, magnetostatics, and time-varying fields. The sequence culminates in Maxwell's equations. In terms of domain-specific problems, we take students through transmission lines, wave propagation in free space, plane wave in multiple media, waveguides, radiation, and antennas.

There are notable variations in the ordering of topics. In the book by Ulaby and Ravaioli [5], transmission lines are presented before electrostatics. Their idea is to make a "bridge" between what entering students are familar with and what they are to study. The book by Johnk[6] has a drastically different approach. It delves into full Maxwell's equations immediately after the concepts of charge density, current density, electric field, and magnetic field are introduced. With this approach, electrostatics and magnetostatics are treated as specialization of the general equations.

Regardless the ordering, the central task is to teach the classical theory of electricity and magnetism. The entire theory can be packed into a few equations—Maxwell's equations plus Lorentz's force law. One can list them all on one page. From these few lines, all other equations and formulas in the course can be derived. The image of a single page containing entirely the entire course content is, of course, deceptive. We all know how hard it is to teach these few lines of equations.

Why is it so hard? Let us take a simplistic view and consider some numbers. In the text we use, there are 136 numbered equations in the chapter on electrostatics. In the chapter on magnetostatics the number is 106, and in the chapter on Maxwell's equations 96. At the end of each chapter there is a list which summarizes the important terms introduced in the chapter. The terms include basic laws (e.g., Gauss's law, Faraday's law), new concepts (e.g., current density, polarization vector) and terms related to techniques (e.g., Gaussian surface, Ampere contour, image method). The total number of important terms of the three chapters is 96. Adding the chapter of vector calculus, there is a total of 113 important terms and 421 numbered equations. Presumably, to understand Maxwell's equations one needs to study all these items in 10 weeks. This is the perception by the students. They are to meet an extraordinary expectation in a very short time.

This perception has natural impact on what one chooses to do to pass the class. Many students respond to the demanding workload with an approach of formulaic learning. We are not alone in noticing a disappointing outcome. Some students who pass the course did not really learn the concepts. What do we do to change the undesirable outcome? Does the course cover too much? Should more time be given to theory and less to applications? Or, should it be the reverse? These are reasonable questions to ask. But this paper takes a different path to consider the problem.

We turn our attention to the process of learning. A learner's behavior can be influenced not only by the external demands, but also by the internal interests and motivations. We think that insights into the internal process could inform us how to teach. In the next section we delve in the theory of cognition process of learning and focus on the concept of mental imagery.

3 Mental Imagery in Concept Learning

3.1 Theory of Mental Imagery in Cognition Research

The concept of mental imagery has been well-established in the field of cognitive science. It is widely accepted that human thinking involves non-verbal elements[7], namely, mental images. The term images here is not limited to visual imagery. It can include, for example, imagery associated with hearing. Pavio proposes the Dual-Coding Theory[8] to characterize the process of cognition. Unlike linguistic elements, the mental images are not well structured and hard to describe. Nevertheless, they are necessary in conceptualization and problem solving. Some researchers think that non-verbal imagery comes before the linguistic process. In their view, mental imagery serve as a basis for a concept to be described in structured languages. According to Rudolf Arnheim [9],

> "purely verbal thinking is the prototype of thoughtless thinking… it is useful but sterile. What makes language are the concepts to which words refer. The concept themselves are perceptual images and the operations of thought are the handling of those images."

According to this view, imagery dominates over language, and plays a role of mediation for a learner to acquire a new concept.

3.2 Personal Testimonies

That imagery plays a dominant role in conceptual thinking is widely confirmed by scientists and engineers through their self reflections. Albert Einstein described his own thought process[10] in the following statement:

“Words or language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be voluntarily reproduced and combined ... this combinatory play seems to be the essential feature in productive thought—before there is any connection with logical construction in words and other kinds of signs which can be communicated to others ...”

In his second volume of *Lectures on Physics*[11], having developed Maxwell’s equations, Richard Feynman has an entire section on Scientific Imagination:

“I have asked you to imagine these electric and magnetic fields. What do you do? Do you know how?... I’ll tell you what I see. I see some kind of vague shadowy, wiggling lines–here and there is an **E** and **B** written on them somehow ... an arrow here or there. ”

“Our science makes terrific demands on the imagination ... when I talk about the electromagnetic field in space, I see some kind of a superposition of all the diagrams which I’ve ever seen drawn about them.”

Images and imagination are no less crucial to the field of engineering. In the book, *Engineering and the Mind’s Eye,* Eugene Ferguson traces the history of engineering creativity. He argued that much of engineering inventions and tool development in past 500 years are the fruits of “non-verbal learning and non-verbal understanding.”[12]

3.3 Direct Observations of Mental Images in Problem Solving

We can find evidence of mental imagery with a learner’s writings and drawings. Qin and Simon[13] conducted a controlled observation to examine the usage of mental imagery in concept learning. The subjects were asked to read and to understand Part I of Einstein’s 1905 paper: *On the Electrodynamics of Moving Bodies*—the famous paper on special relativity. The paper is completely written in language and mathematics. There are no drawings or figures. From the study, Qi and Simon concluded that the subjects created drawings and mental representations to “mediate between the natural language text and the final equations. In no case did any subject achieve an understanding of the equations without using this kind of intermediate representation."

We made an uncontrolled observation in our fields class, and we reach a similar conclusion. A class was given the following problem in an exam:

Problem

Two long, parallel wires are 6m apart. They carry steady currents of 10A each, in opposite directions. Place a third long wire in parallel with the two wires, at a distance of 5m from each of the two wires. If we run a steady current of 10A through the third wire, determine the force per unit length acting on the third wire by the other two wires. Use drawing to present your analysis. Make clear the directions of the currents and the force with these three long and parallel wires.

No additional information—drawings, figures, or formulas—is provided. There are 57 submitted solutions. We examined three aspects of each solution: a) Does it use drawings? Is the drawing correct? b) Does it arrive at valid mathematical equations? c) Does it find the correct answer? We group the results into five categories in terms of these three aspects. Our finding of the 57 submissions is tabulated below:

Category	Description	Population
I	valid drawing; valid equations; correct answers	29
II	valid drawing; minor errors in equations; incorrect answers	3
III	invalid drawing; invalid equations; incorrect answers	6
IV	no drawing; invalid equations; incorrect answers	15
V	no drawing; valid equations; correct answers	4

As the analysis shows, there is a strong correlation between making valid drawings, finding valid equations, and arriving at correct answers. By and large, invalid drawings or no drawings at all lead to incorrect answers. Only 4 out of 57 somehow find valid equations and correct answers without showing evidence of non-verbal imagery.

To us, this finding is not a surprise. It confirms the view that drawings are a crucial mediator in engineering problem solving. Whereas no figures are provided, the images are produced through a self-discovery process. The observations made by Qin and Simon and in our class give strong credence to the theory that mental representations are crucial to the process of concept learning and problem solving.

3.4 A proposal for teaching electromagnetism

Based on the argument above, we accept the premise that mental imagery inevitably forms in all experiences of concept learning. Thus, we propose the following pedagogy principle for the fields course:

Teaching materials and methods should help students develop effective mental images of the concepts in Maxwell's equations.

4 Getting the Concept vs. Matching the Formula

The "gap problem" mentioned earlier reflects a common tendency among our students. Confronted with a problem, the first reaction is to find a matching formula. We observe that the fields course seems to exacerbate such a mental habit, especially in the second half of the course.

The second half of the course is about applying Maxwell's equations to domain-specific problems, such as wave propagation in free space, plane wave propagation in multi-meida, transmission lines, waveguides, etc. A common approach is used in many textbooks to treat these applications. First, a general problem is described with its domain-specific features and boundary conditions. Second, the theory of Maxwell's equations are applied to derive equations and formulas that constitute the solutions to the problem. Then, the utility of the formulas is demonstrated with exercises.

The second step in this approach tends to be dry and tedious. Students often skip it. They simply focus on the formulas and see the learning as to follow the procedure with formulas. It is not uncommon that a student studies an entire chapter on an applied topic without a revisit to the basic theory. This is unfortunate. Great opportunities for learning the concept through application are lost. Let us use a specific example to illustrate this point.

Consider the problem of an electromagnetic plane wave arriving at a boundary between two media at an oblique angle. There is an incident wave, a transmitted wave, and a reflected wave. A standard treatment in a text would derive two sets of 6 equations; one for parallel polarization and another for perpendicular polarization. The equations are expressed in terms of intermediate variables such as the angles of the waves, the coefficients of transmission and reflection, etc. To derive the equations, it requires real understanding of Maxwell's equations. Each pair of the electric and magnetic fields of a wave must satisfy the curl equations. The three waves must satisfy the boundary conditions, and they are also determined by Maxwell's equations.

After the derivation of the formulas, problems for exercise follow. A typical problem is like this: Given an expression of the incident wave, find the reflected wave and the transmitted wave. Adopting a formulaic learning strategy, one may do the following. A student may go directly to the formulas, the 12 equations. Identify the polarization, and choose a corresponding formula. Then, start a process of computations. The process involves many details and is prone to mistakes. We often see that the end result from a student contains wrong signs, uses mismatched equations, or gets the directions of fields wrong. Students are supposed to self check their results at the end. There are many details to check and they may simply go over the formula again. The whole process can be tedious and very frustrating. It is even more frustrating, if one does not understand or cannot recall the concepts that underlies the formula. Not being able to see the concept of curl in the wave expressions, a student may see the entire effort as a mechanical process or find it meaningless.

The concept of curl is difficult for many students. Throughout the course, the operation of curl may not get enough practice for the concept to sink in. Often, one has forgotten the formula for curl by the time the second half of the course begins.

If we follow the pedagogy principle proposed in the previous section, what would we do differently? What can we do to help students develop their mental images about the concept of curl? First, let us recall the general idea. Verbal descriptions and mathematical representations alone are not sufficient. To be added is a continued prodding with questions and problems for imagination and for practice of qualitative reasoning.

Here we give an illustration of what we mean by an imagery-based approach. We use a sequence of images, Figures 1-3, to describe a process of inquiry. When curl of vector fields is first introduced, students are presented with Figure 1., which is adopted from Purcell and Morin[14]. We ask the students to "see" the curl of the vector field. We may also suggest using line integrals at various points as a way to see. Figure 2. will be presented when students study traveling waves of fields. They would consider the field as a vector function of both time and space. As a vector field, we again ask them to visualize the curl of the field. Students finally arrive at Maxwell's equations. They would now be asked to see the relationships between two traveling waves, such as the waves in Figure 3. This is a challenging task. It may turn the entire class into a community in search of pictorial representation of Maxwell's curl equations. The search, even if it does not result in a clear or complete picture may give tremendous support in the study of applied problems such as the waves in two media discussed earlier. The imagination effort may enable the students to see the complex formulas with more discerning eyes.

The figures are presented in a qualitative manner, with only sketchy quantitative aspects. The drawings lack a degree of specificity for numerical calculation or computer simulation. This is intentional. It is aimed for the development of ability to visualize.

5 Mental Images and Formal Graphic Tools

The idea of teaching concepts with imagery is not new. In many fields, including engineering and physics, graphic tools have been widely used to enhance communication and learning of concepts. Used in teaching, graphics and diagrams can strongly influence the forming of mental imagery of concepts. But, are mental imagery and graphic tools the same thing? We argue that they are not. In the following we use the pedagogy proposed by Van Heuvelen[15] to illustrate the difference.

Van Heuvelen proposed a pedagogy for students to focus more on physics concepts than on formulas—the same challenge as ours. The process comprises a sequence of four steps: a) describe a problem in words, b) construct pictorial representations, c) construct physical representations, and d) find quantitative solution with mathematical representations. The steps of making pictorial and physical representations are inserted between describing a problem in words and finding solution with mathematics. The purpose is to enhance concept learning with pictorial and qualitative reasoning. But, as we point out earlier, mental imagery develops through all channels of perception and in any thinking process. Conceptual imagery inside a learner emerges as soon as one reads the word description. The imagery continues to evolve in other steps of the process. The act of constructing pictorial representations may greatly influence the forming of mental imagery, but the formation is not exclusively the outcome of pictorial construction. The point is that mental images and pictorial representations are two different entities. The former is internal and subjective; the latter external and objective. We can reflect on a common observation to see this point. When students are taught with a same process and the same pictorial representations, they often develop different views of the concept at the end.

Other aspects of mental imagery are also important. Mental imagery development process takes time and may continue to evolve even after formal learning process ends. The notion of a continual process is well supported by cognitive science research. It is interesting to note that Van Heuvelen also touched on this point in the paper. About why students resist using pictorial and qualitative reasoning, he said: “the student mind holds many preconceptions that have been stored numerous times during 20 or more years... ” What he described here is a scenario in which mental images, the “preconceptions”, can hold power for many years. And, the images can influence and interfere with present learning even though they were formed through other learning experiences long ago.

We think that to meet our challenge it takes more than teaching students graphical tools. We plan to comprehensively incorporate the concept of mental imagery into our teaching methods and materials in the fields course. Our future work will take into account the following considerations:

1. Though it is difficult (or impossible) to fully know a student’s mental imagery, there are many tangible ways to “estimate” it. Drawings, written words, patterns of errors, all reveal much of what is on one’s mind.

2. The internal imagery development is intrinsically a process of discovery. The teacher provides external stimuli, but it is the student who does the work to form the images. The teacher should aim for independent work by the student as much as possible.

3. To help form effective mental images, there are many tools other than formal drawings. Word description, verbal articulation, problem solving, lab demonstration, quiz questions, graphical and even video images can all be effective stimuli.

4. Like all mental developmental processes, internal image development takes time and continuation. Frequent visits to the same concept in different settings—a point highlighted in the pedagogy by Van Heuvelen—is a good strategy, which is also a factor in our example in Section 4.

6 Conclusion

In summary, this paper makes a case for this pedagogy principle: Teaching electromagnetics should recognize the tendency of students focusing formula and should help students develop effective mental images and real grasp of the concepts.

Maxwell's equations are a sophisticated mathematical construct of concepts and relationships. The theory is built on basic elements of charges, currents, electric fields, and magnetic fields. The concept of fields and waves cannot be really understood outside the equations; they are intrinsically *defined by* the equations. The inseparability between the concepts and mathematical equations makes the study difficult. But, to have a real grasp of the concepts, there is no going around the mathematics. That is why we argue for the crucial task of forming effective mental imagery. With our future work we plan to contribute to the pedagogy literature specific design of teaching materials and their assessment. Hopefully, they would produce better learning outcomes.

7 Acknowledgment

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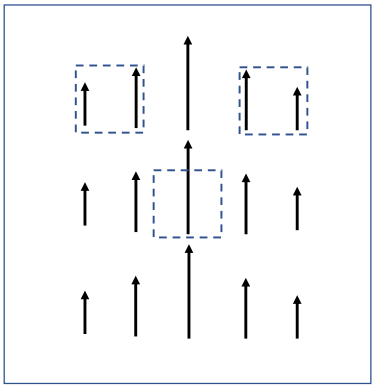

Figure 1. Curl of a Vector Field

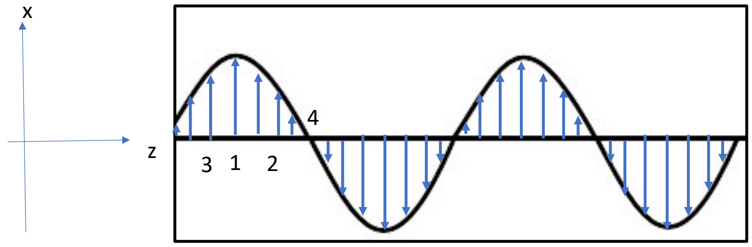

Figure 2. Space- and Time-Derivatives of a Traveling Wave

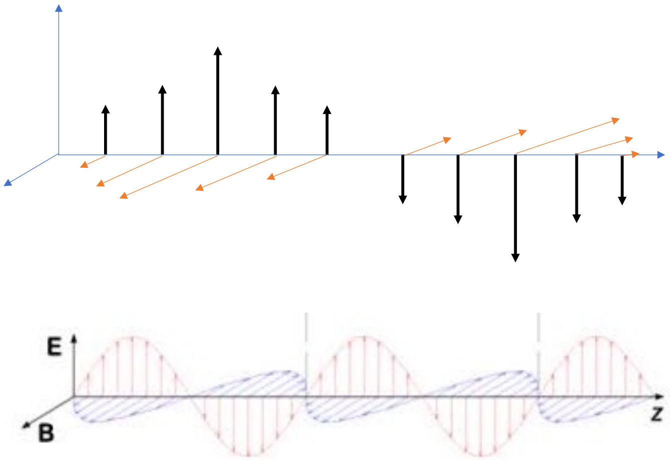

Figure 3. Relationships between Two Waves