# Sustainable Collaboration Paradigms Between Math and Engineering

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# Work-in-Progress: Connecting Engineering with Mathematics through Differential Equations

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## Abstract

**The work described in this paper is part of a larger, collaborative NSF grant. The focus of the grant is to study faculty attitudes, culture, and protocols needed to establish successful cross-campus connections between mathematics faculty and partner disciplines. The intent is to develop a systematic approach to collaborative education that is independent of factors such as school size, topic emphasis, and the variety of partner disciplines, to name a few. The larger team consists of both four and two-year institutions that range in size and type from small, private with specific focus areas to large, public with a wide variety of disciplines. VCU is a large university (~30,000 students), offering 200+ programs. For this project, the chosen partner disciplines are Biology, Chemistry, Art, Business and Engineering. The focus of this paper is the development of the partnership between Math and Engineering and how this partnership led to our approach to expand and deepen the relationships between Math and the other disciplines while at the same time improving student and faculty engagement, leading to better student outcomes.**

## Keywords

Mathematics, Engineering, collaboration, 21st century skills, engagement, learning

## Background

At VCU, the math department teaches an average of 11 sections of MATH 151 (Pre-Calculus), 25 sections of MATH 200 (Calculus I), 12 sections of MATH 201 (Calculus II) and 9 sections of MATH 301(Differential equations) per semester. Students in these sections include majors from Math and partner disciplines ranging from the School of the Arts to Humanities and Science, Business, and Engineering. At the onset of the project, the two PIs (one from Math and one from Engineering) conducted a faculty survey to gauge current attitudes, level of interest in the project, and the degree to which faculty felt the math courses listed above impacted their students' success. Broadly, the survey showed that engineering faculty were interested in how mathematics courses were (or should be) preparing students for engineering courses and that lower level skills (i.e. algebra) were a source of weakness.

The ultimate goal is to develop a transferable, sustainable and impactful roadmap to curriculum collaborations with all the partner disciplines. Before the start of the grant, there were small, ad hoc, often accreditation related, conversations about course content. Ideally, we hope to establish a systematic schedule for classroom visits, curriculum conversation meetings, quantitative assessment of student transfer of knowledge, and qualitative assessment of faculty and student attitudes of preparedness. After the initial outreach to all the partner disciplines, the team decided to start small, concentrating on the Differential Equations course, which is comprised of 75-80%

engineering students (98% engineering and science students), for whom the discipline-specific examples, in-class worksheets and assignments being developed would be equally relevant. Anecdotally, and as this is articulated through the course assessments performed by faculty for accreditation purposes, students do not do a good job of transferring knowledge content or acquired skills from this math course into courses within their majors. In order to improve knowledge and skill retention, we have worked to establish systematic collaborations between Mathematics and the partner disciplines, primarily Engineering. In addition, we communicate regularly with our cohorts at the other institutions on the collaborative project to review different collaboration paradigms and determine a qualitative process or list of best practices for setting up similar collaborations in diverse settings. The purpose of the specific collaboration between Engineering and Mathematics at our institution is to:

1. Investigate pre-existing faculty biases on why students found it difficult to transfer knowledge between different fields;

2. Develop a systematic approach to collaboration between the two departments that will lead to a better understanding of the difficulties faced by our students and thus to continuous improvement of both the Engineering and the Differential Equations courses;

3. Provide a "roadmap" that will enable other disciplines within VCU and other universities to develop similar collaborations between their Math and Partner Disciplines.

Using the "Curriculum Foundations Project: Voices of the Partner Disciplines" report [1] as a source of discussion questions, a Fishbowl activity was held with faculty from Chemistry, Biology, Physics, and Engineering. This activity led to a better understanding of what the partner disciplines could and could not expect their students to know and be able to do once they completed the Differential Equations course. The Fishbowl discussions led to follow-up activities, such as prioritizing the course content and co-developing application problems and projects so students could understand how the differential equations course is applied in their field of study. [2][3][4]

The VCU project leaders also held frequent meetings to determine their strategy going forward. The particular challenges at this institution were the large number of partner disciplines that were impacted by the courses, the large class size, especially in the pre-Calc to Calc II sequences, and the emphasis of the university on having all students take a "Core" math course, which will be the same regardless of the discipline. The main idea examined was whether the team could develop a teaching structure within the mathematics courses that incorporated discipline-specific examples, homework and worksheets that would help increase students' sense of relevance for what they were learning as well as serve as mnemonics for when they were faced with similar (albeit more in-depth) problems later. [5]

The outcome of the discussions was that Math would focus on developing a template for MATH 301 (the Differential Equations course), a three credit, primarily lecture course, taught by tenure-track faculty, term faculty, and visiting faculty. This course has a high concentration of engineering students (an average of 75% across all sections). In this way, math could focus on developing material for one content area as well as developing a systematic approach for collaborating with one partner discipline that could be expanded to include more disciplines later. In addition, the team feels that assessment of the outcomes will be more straightforward. An essential component of this project is its continuity and emphasis on the continued collaboration between Math and Engineering. To that end, the Math department hosted a general interest meeting for all interested faculty where their instructional paradigms and topic emphases were discussed and the partner discipline faculty had the option of observing a Math class. Feedback from the meeting was provided through conversations during the meeting and in follow-up surveys. These activities mark a formalization of interactions between the math and engineering departments.

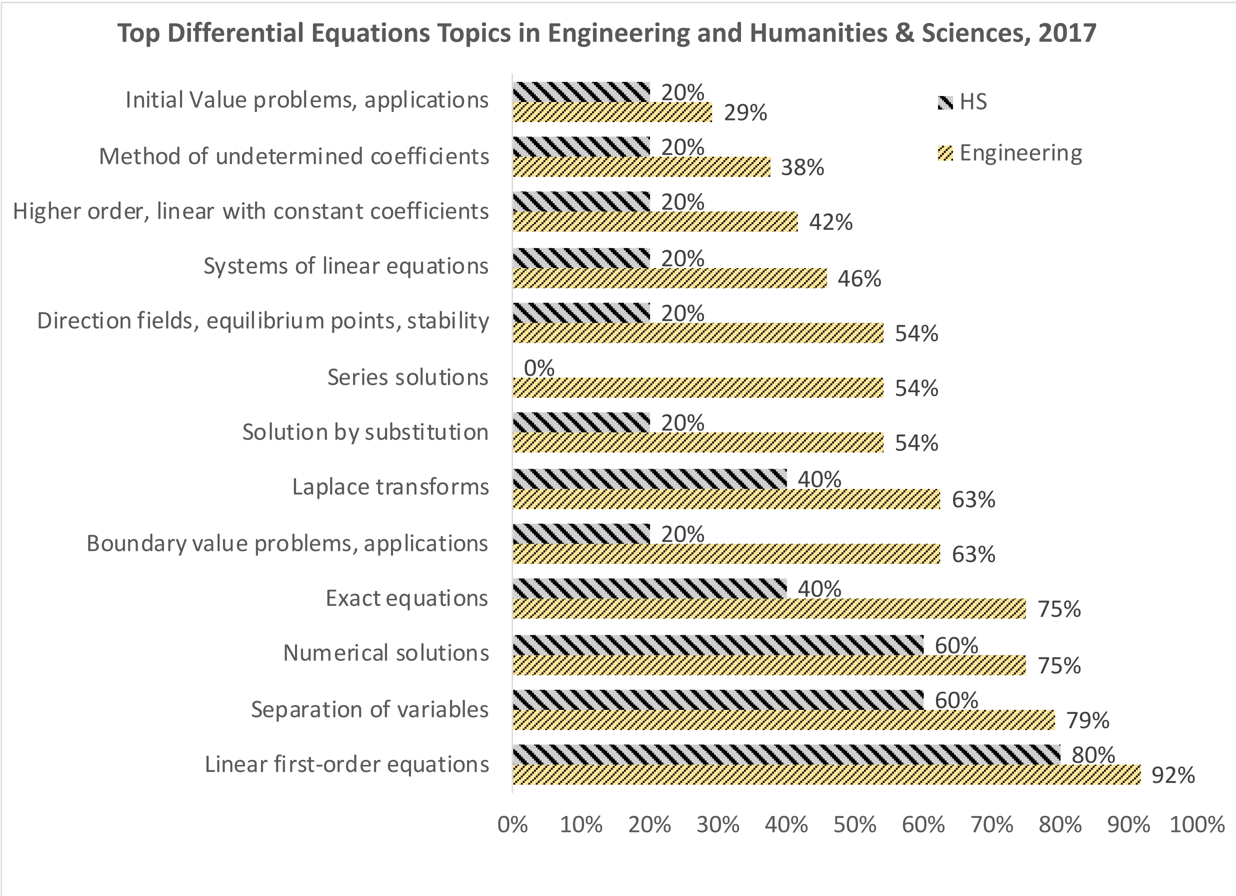
In the larger collaboration effort, the PI and co-PI of the grant visit two partner institutions to observe their activities and how they have implemented their collaborations between Math and their partner disciplines. The partner institutions are diverse and include public, private, HBCU, large and small Universities and one Community College. The purpose of these meetings is to examine similarities and differences in the collaboration paradigms and educational improvements and to assess the applicability of these paradigms to other institutions. The goal is not to create a master list of best practices for every institution, but to create a flexible prototype for helping any university improve inter-disciplinary communication.

The authors present survey data results collected through student and faculty surveys, as well as an outline of how they intend to continue the collaboration beyond the scope of the grant. Preliminary results are presented, along with details about the work going on at our institution, outlining successes, challenges and sustainability and applicability to math courses other than Differential Equations.

**Project Onset:**

**Survey Spring 2017:** A survey was conducted in the spring of 2017 that focused on MATH 251 (Pre-Calculus), MATH 200 (Calculus I), MATH 201 (Calculus II) and MATH 301(Differential equations). The goal of the survey was to engage partner discipline faculty in the discussion and to receive specific feedback on what they are observing in the classroom, but also the degree to which the current topics covered in each math class train the students adequately for their classes. Thirty-eight faculty members responded to the survey, with 29 (76%) of those being from engineering (this corresponds to a 39% response rate from engineering faculty). Since the emphasis of this paper is on the Differential Equations course, only the results from those survey questions will be presented here.

The survey questions were meant to gauge the level to which the current topics covered in the Differential Equations course were emphasized adequately or perhaps too much, depending on the specific area of expertise required by students in subsequent classes. Of the 38 faculty who responded to the survey, 29 responded with feedback specific to Differential Equations. Of those, 24 were in Engineering and five were in H&S. Only the responses from these faculty members were counted. The topic areas under consideration as well as the responses can be found in Graph 1. As can be seen in this graph, engineering and humanities and sciences place proportionate degrees of emphasis on the top-rated topics. This is desirable, since at the time, VCU was not considering teaching separate sections for engineering. The results from this survey were shared with the partner discipline faculty in the fishbowl activity (see next section) and used to determine the emphasis placed on the topic areas currently taught in Differential Equations.




*Graph 1: Faculty survey, Spring 2017. Question pertained to the topics currently covered in the Differential Equations course and their perceived level of importance by the respondents.*

**Fishbowl, Summer 2017:** In the summer, the team used the results of the survey to host a "Fishbowl" activity [1],[6], where they discussed the results with the engineering faculty and gained more detailed information on what the baseline course content should be. Math subsequently cut a few sections from the syllabus and they meet with the partner disciplines every semester to discuss progress, further specify the emphasis on applications and what that means in the context of a math course aimed at students who have not had substantive training in engineering yet.

**Math-Engineering Summit, Fall 2017:** In the fall of 2017, the Math department hosted a meeting with engineering, where all engineering faculty were invited to attend information sessions as well as observe the first implementation of discipline-specific applications in the math classrooms.

Some key points made either during the Fishbowl or after the classroom observations by the engineering faculty addressed the need to incorporate practice in students' parallel math skills, such as reading graphs, visualizing 3-D space, and manipulating functions. In fact, a significant emphasis was placed on the fact that students were most often tripped up by a lack of algebra skills than an inability to set up the solution of a simple differential equation. There has consequently been significant discussion about the need for continuous and as-needed review of the necessary collateral skills students might need in a course. Many engineering faculty have responded by increasing opportunities for students to review these skills at the beginning of their courses.

In addition, there was a significant amount of discussion around the differences in terminology as well as typical notation preferred by the different disciplines. A suggestion was proposed to introduce a diversity of symbols that are more related to the partner disciplines, so students acclimate to recognizing the functional form in its many manifestations.

Other changes targeting a more uniform approach to teaching this course included creating a Blackboard site shared by all course instructors (common syllabus, shared resources); common exam questions were introduced in the fall of 2018.

**PI Meetings:** The PIs for this project met at least once a month and communicated regularly on the results of their activities. This is consistent with the organization of the collaboration between math and partner disciplines at the other institutions, but each university had autonomy in terms of the specific dynamics of the collaboration. Generally, the PIs are in various positions within their departments and universities, but all PIs received letters from their department chairs and deans supporting the efforts of the grant and supporting curriculum development resulting from the grant activities.

At VCU, the purpose of the meetings was to manage expectations, determine individual responsibilities, stay on task and maintain a sense of urgency for the project. Math PI responsibilities included project oversight and final say on changes the math department would be able to implement within each course. Engineering PI responsibilities included engaging engineering faculty through the surveys, performing the data analysis on the surveys, and more recently, defining (in collaboration with engineering faculty) the assessment mechanisms to be used to evaluate student outcomes.

In addition, the meetings served as brain storming sessions, where the PIs regularly reviewed the results of their collective effort and adjusted their course of actions accordingly.

**Visits to other project locations:** In order to further invite collaboration and incubate new ideas, the team also hosted collaborators from other universities and visited other universities to learn about their initiatives, processes and resulting action items. The visits were meant to be between a variety of institutions, some of which resembled each other and others that were significantly different. The idea is to learn from each other and adapt ideas that might need to be modified to fit a certain setting, but still provide an impetus to think creatively.

These sessions, which other interested faculty were invited to attend, led to the current initiative, which is to use LONCAPA (Learning Online Network with Computer-Assisted Personalized Approach – e-learning platform [7]) to develop a range of online, dynamically allocated exercises that the students and faculty would have access to for assessment, review, practice and grading purposes. The team is currently engaged in developing learning modules to be used by both math and engineering faculty. These modules are relevant to specific topics in engineering and have “Preview” sections and “Review” section. Course modules will be made available through LONCAPA’s open access platform.

The “Preview” modules are meant to be used by Math faculty to provide students with discipline-specific applications for the math they are learning; the goal here is to set up the problem for the student with a meaningful reference and topic-specific variables and constants but with the emphasis on the techniques the students would use to solve the problems. On the other hand, the “Review” modules are meant to be used by the faculty who are teaching students an engineering topic that requires certain math skills that will be used to either set up or finalize a solution. This provides a continuity of instruction and knowledge and skills development that is crucial for a learner [3],[4],[5].

The idea originated from a visit to Augsburg University, a small, private university with small class size and a diverse population. The partner disciplines in this case were chemistry and economics. The faculty there had implemented a hands-on approach in the classroom that seemed to be uniquely suited to that environment. The VCU team, however, is from a large, metropolitan, diverse university where math classes can be large or taught by multiple math faculty, each with their own style. Thus, VCU is taking advantage of technology to implement a similar system in their larger classrooms and expand it to engineering. In addition, the math faculty is adapting their in-class instruction to see if the Augsburg model can be implemented in their larger classrooms, while the engineering faculty has adopted the model with only minor changes in their small engineering classes.

In addition to the annual site visits, there were virtual meetings every other month with PIs who were teaching similar courses at their institutions. Challenges and changes made to curriculum in Calculus and Differential Equations were discussed regularly.

Results:

The research team is relying on external evaluators to ensure the project is meeting expectations and to evaluate the overall efficacy of the team-building and sustaining efforts, the portability of the collaboration principles established at each institution and the impact on the faculty and students in all institutions. The VCU team is setting up and has started to evaluate on a more local scale the sustainability of their efforts: (1) in terms of establishing, maintaining and expanding the collaboration between the faculty in Math and Engineering and the degree to which this is translatable to other partner disciplines and (2) in terms of student outcomes.

To do this, the team is designing and implementing qualitative and quantitative assessment protocols. The qualitative assessments are in the form of faculty and student surveys to gauge: (for faculty) (1) understanding and appreciation of the goals of the collaboration, (2) perceived impact of the project (for students) (1) awareness of the presentation of practical applications in the math classes (2) perceptions and self-assessment of their retention of the topics learned in the math classes and their ability to apply them to their engineering classes.

The quantitative assessments will track students as they complete their mathematics course and will attempt to gauge whether their efficacy in math is improving. This will be done through tracking them in future courses in engineering that require mathematics. The tracking will be performed in collaboration with faculty, who will implement assessment mechanisms to gauge whether the students, as a whole, are improving in their ability to apply the requisite math skills to their engineering classes.

At this time, the cohort of students who have had the benefit of the new math classes have not all taken a correlated engineering course, so the team is working on establishing baselines and substantive tracking mechanisms. The mechanisms for establishing baselines will include:

1. A survey of current ABET assessment of student math skills, separated under the relevant math topics being examined. Reports from the previous two ABET cycles will be utilized.
2. A set of new questions will be added to the current assessment mechanisms:
    a. Original questions will provide a map to prior student achievement;
    b. New questions will capture more in-depth understanding of collateral skill achievement, such as algebra.
3. The 2017 and 2019 surveys of students and faculty, to gauge attitudes towards math will be used as a baseline and further deployed to track changes in attitudes.

**Faculty survey 2019**: In order to assess faculty awareness of and sense of inclusion in this project, we plan on running regular surveys to gauge continued need for the topics identified from prior surveys (in this case, the one in 2017, with results shown in **Error! Reference source not found.**Graph 2, or to identify new needs. In addition, this will operate as a climate survey to gauge faculty satisfaction with the results.

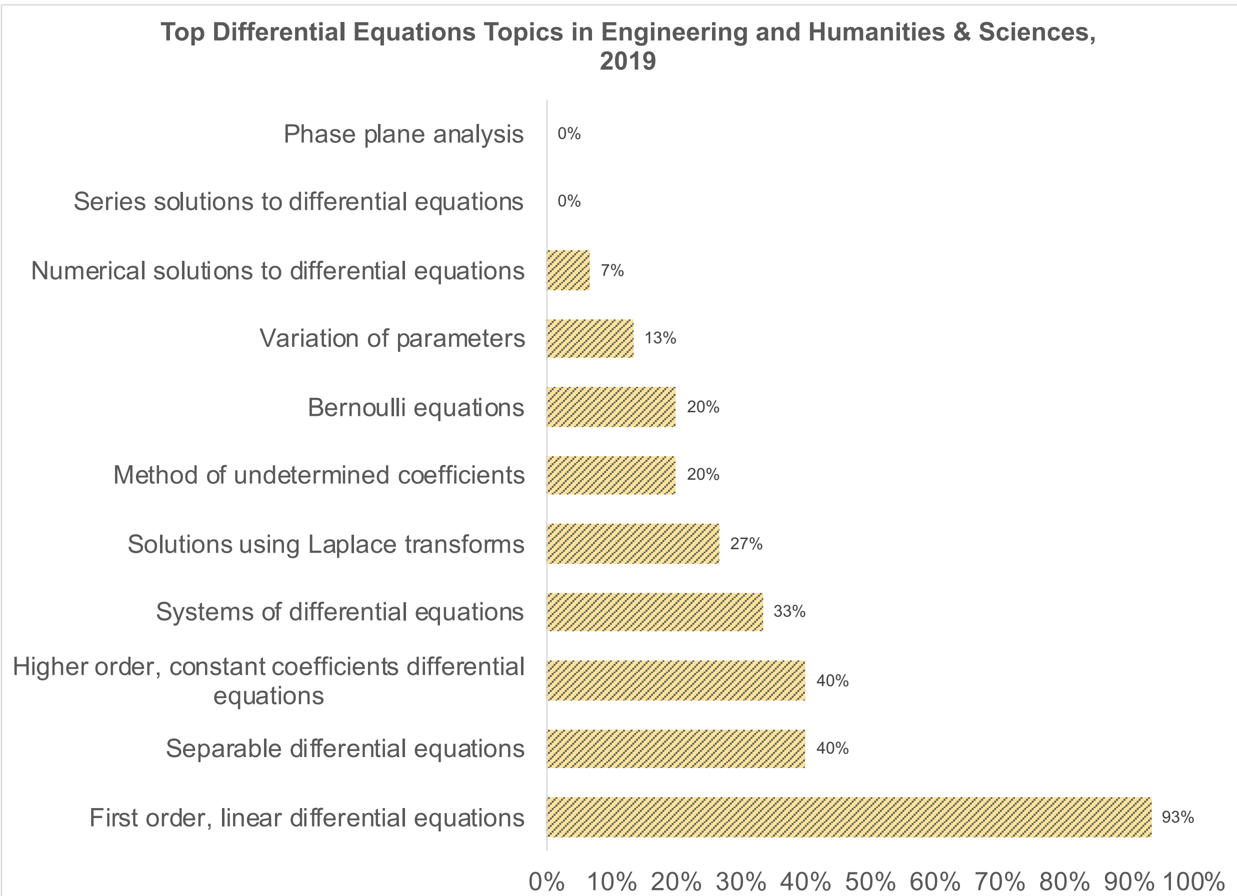
The 2019 faculty survey was sent out on 11/15/2019; the response rate was 45% (30 faculty) from a mix of disciplines. On this occasion, it was anonymous. Of the 29 respondents, 14 (48%) claimed to be teaching classes that require differential equation concepts. Only seven entered specific courses; they were in all disciplines offered at the College (Electrical and Computer, Chemical, Mechanical and Biomedical) except for Computer Science (Differential Equations is not required in CS). Of those that responded, 14 claimed to teach a course that required Differential equations, while 15 said they did not need Differential Equations in their courses. The following are the answers provided by the faculty that taught classes requiring Differential Equations.

**Question 1:** What (differential equation) topics are relevant and used in your (engineering) undergraduate courses?

This question was a “conditional” one, based on the respondents’ answer to question 1; the surveyed faculty would only access that question if they stated that they taught a class requiring concepts from MATH 301.

The results for this question are shown in Graph 2. In this survey, a different set of topics are emphasized, that were deemed to be more relevant to engineering faculty. Linear first-order equations continue to be the highest chosen category, as are separable differential equations and higher order differential equations (earning 40% in both surveys). Some new categories are phase plane analysis and series solutions, that did not generate any response; numerical solutions had a 7% response, possibly because the disciplines that need this topic are currently teaching it themselves. This could also explain the decreasing importance placed on Laplace transform solutions. This question on the survey will continue to be used as a "continuous improvement" mechanism for all math courses to evaluate timeliness and relevance of the math courses being taught.

In the future, the surveys should include all the original questions, with only the low-placing questions being cycled out in subsequent surveys.

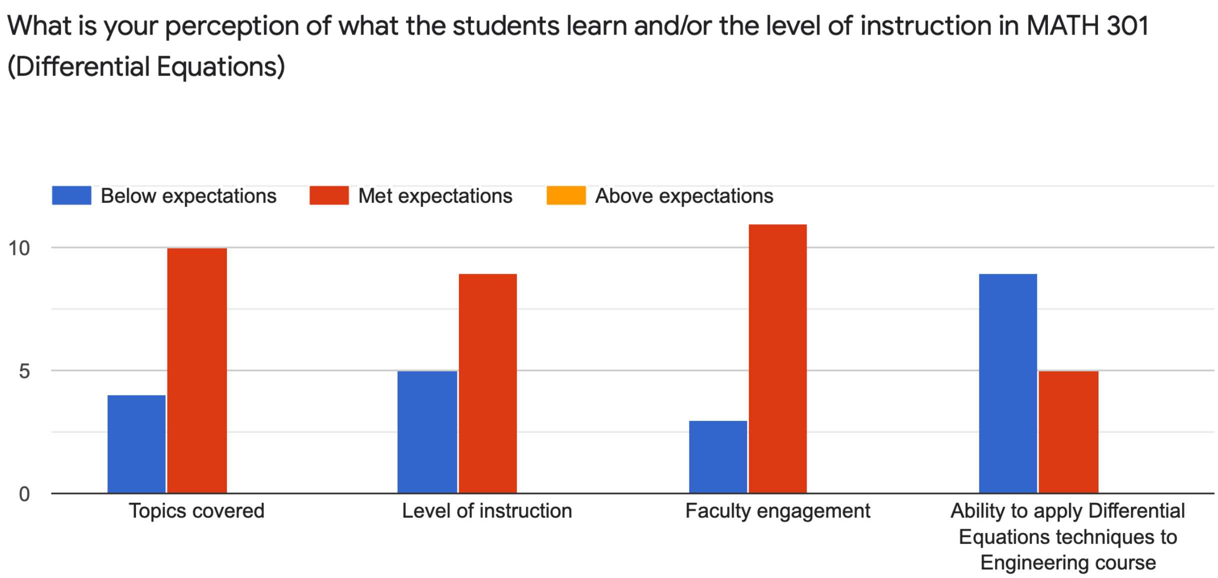



*Graph 2: Faculty survey, Fall 2019. Question pertained to the topics currently covered in the Differential Equations course and their perceived importance by the respondents. There is a variation between these topics and those surveyed in 2017; this reflects the desire of the team to offer some other topics not covered in 2017.*

**Question 2**: What is your perception of what the students learn and/or the level of instruction in MATH 301 (Differential Equations – in terms of topics covered) (Conditional on answering "yes" to Question 1 above).

Fourteen people answered this question; the results can be seen in Graph 3. As can be noted, no engineering faculty chose "Exceeded Expectations", thus confirming the necessity of this initiative, but 71% claimed that the course and/or student achievement "met" their expectations. This question might need to be separated into two parts – one relating to students and the other relating to math faculty – as well as a separate question trying to gauge what engineering faculty's expectations are. However, a 71% "met expectations" seems to indicate that most students are performing adequately, but there is room for improvement.

In the future, this question will be open to all respondents, as we can use their answer to question 1 as a filter. In addition, a separate question should gauge people's expectations, as the answer to that question is very subjective.




*Graph 3: Faculty perceptions of what engineering students learned and/or the level of instruction in Differential Equations.*

**Questions 3-7:** Engineering faculty interactions with math faculty

Of the faculty who responded to the survey, two had met with mathematics faculty (Q.3). Both had attended the special meeting we held in the spring of 2017 and had participated in the classroom observation opportunity, and one of them had also participated in one-on-one meetings with math faculty (Q.4). In both cases, these meetings only changed their perceptions on faculty engagement (Q.5). The interaction that was listed as being the most impactful was the meeting/classroom observation, but the one-on-one visits also ranked high on the list (Q.6). It does seem, though, that building in opportunities for faculty socialization and active exchange of ideas is important.

**Questions 8:** Have you provided feedback or input to content for the differential equations course?

Four respondents replied in the affirmative to this question. It is worth noting that 6 faculty members provided example problems in 2017, but not all of these professors responded to the 2019 survey, as they consider themselves part of the team (their feedback later). In subsequent surveys, it will be made clear that we need feedback from everyone. The faculty who have engaged with this effort include tenured, tenure-track and non-tenure track faculty. The distinguishing factor is their perception of the importance of mathematics to the course that they teach and their sense of personal responsibility to do what they can to improve outcomes for their students.

**Question 9:** Do you perceive any difference in student performance relative to math background in the last year?

Of the four respondents, the subsequent question of whether this seems to have brought on a change in student performance, two said "No noticeable change", one said student performance had improved and one said it had declined.

**Question 10:** Comments

In the general comments section, only three professors responded. One decried the students' inability to do mental math (this was a professor who does not require Differential Equations in their course), one identified their lack of ability to do algebra (this is an outcome from our observations as well) and cited that as a reason to not blame the math instructors. This comment was followed, however, by the assertion that VCU had "lowered its admission standards". This is factually not true and opens up an interesting avenue of study in the area of faculty attitudes. The third comment was that students seem to be able to do the steps to solve a differential equation but do not understand how to interpret a differential equation and its solution. This comment, also, opens up an interesting avenue of study or opportunity for collaboration in terms of investigating at what point students are equipped to understand the contextual aspects of any mathematics course and who should best take on the burden of that lesson. It also highlights the value of reviewing concepts in class, where a specific professor can put things in the context that is specifically relevant to what they are studying.

This survey should be repeated at regular intervals, after the initiative has been implemented for a while and definitely after it has been implemented in the Calculus classes. Regular feedback from partner disciplines is necessary so that examples and practical applications remain current.

**Student Survey:**

Student surveys were collected under IRB approval (IRB HM20007626) and were conducted without collecting identifying information about the students.

**Question 1:** How much time did the Differential Equations course spend on practical applications?

*Table 1: Results from Fall 2019 student survey: How much time did the math faculty spend on practical applications?*

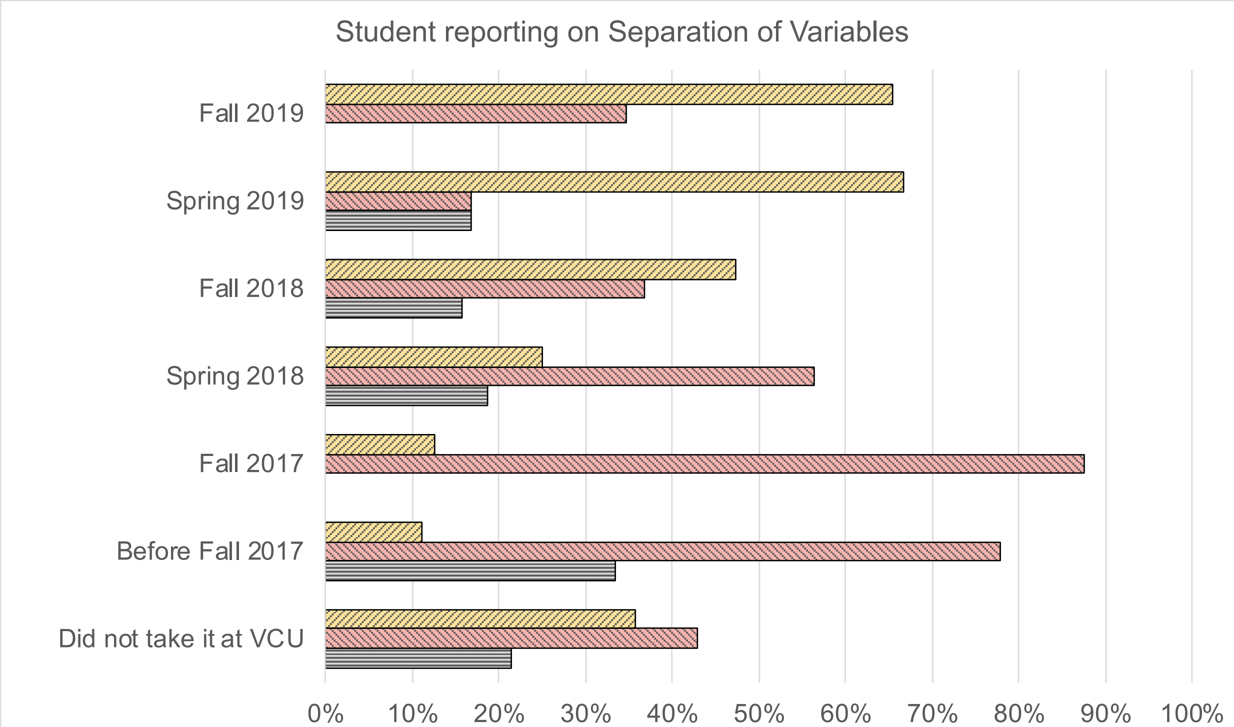
| | Very many | Some | Not many | Total # | Timeline |
|---|---|---|---|---|---|
| Before Fall 2017 | 3% | 47% | 72% | 32 | Onset of award cycle |
| Spring 2017 | No survey participants | | | 0 | First strategic planning meetings between PIs; first survey sent out. |
| Summer 2017 | Fishbowl | | | 0 | Fishbowl activity |
| Fall 2017 | 23% | | 77% | 26 | First implementation of engineering specific examples; not all sections |
| Spring 2018 | 24% | 29% | 48% | 42 | Engineering-specific examples, all sections |
| Fall 2018 | 11% | 32% | 57% | 65 | Common Blackboard site, syllabus, exams |
| Spring 2019 | 19% | 56% | 25% | 16 | |
| Fall 2019 | 31% | 21% | 47% | 70 | |

In answering this question, students do not demonstrate an awareness of the expanded collection of practical applications that has been implemented in their classrooms. This is natural, as these students do not have anything to compare their experience to.

**Question 2:** Think back to your differential equations course. For each of the topics, please rate how much of the topic you remember and how useful it was for your engineering courses.

Two topics will be presented in full detail in Graph 4 and Graph 5, as an example of how we are looking at the data to try to gauge needs for future improvements and then all the topics are summarized in

Separation of variables**Error! Reference source not found.**(see Graph 4). Specifically, this graph shows a marked increase in student's sense of having retained the concept of "separation of variables" and being able to use it in subsequent classes (the question does not ask if they remember this now, but if they remembered it when they needed it in their engineering classes). There is a greater sense of retention in the courses the students took subsequent to 2017.




| | Did not take it at VCU | Before Fall 2017 | Fall 2017 | Spring 2018 | Fall 2018 | Spring 2019 | Fall 2019 |
|---|---|---|---|---|---|---|---|
| Remember well/use often | 36% | 11% | 13% | 25% | 47% | 67% | 65% |
| Remember a little/use some | 43% | 78% | 88% | 56% | 37% | 17% | 35% |
| Do not remember/do not use | 21% | 33% | 0% | 19% | 16% | 17% | 0% |

*Graph 4: Self-reporting of students on their ability to recall/frequency of use of Separation of Variables. In future surveys, these questions need to be asked separately.*

While the results in this survey need to be further examined, at least as regards to the topic of "Separation of variables", which was listed as second in order of importance in the 2017 survey (see Graph 1), there is a promising trend upward in the "Remember well/use often" category. The Laplace transform, which was listed fifth, does not show a similar trend. From interviewing the faculty in Electrical and Computer Engineering, there is anecdotal evidence that this is due more to the fact that they are de-emphasizing the use of the Laplace transform in the classes (such as Circuits I and II) where it would be most prevalent than to any changes in math instruction. This is an important outcome for assessment purposes and will possibly lead to a correction in the way Circuits has evolved over the years.

The remainder of the results are aggregated (for the purpose of this paper) into one graph showing student perceptions of the topics under examination (see Graph 6) and their ability/tendency to "Remember well/use often". The data in the graphs has a promising trend, but the main outcome is the recognition of the need for joint assessment of math and the linked engineering courses to examine:

1. The degree to which the students are, in fact, asked to apply the concepts they learn in Integrated Circuits in the engineering courses that name this course as a prerequisite;

2. To what degree each concept is necessary;
3. What review activities are the engineering faculty engaged in to help their students recall facts;
4. How much time passes between the students taking the math course and applying the material they are learning in this course;
5. To what degree do collateral skills impact the professors' perception of what the students know and know how to do?

In addition, in subsequent surveys, the questions of how much the student remembers and how much the concepts were used in the class need to be separated.

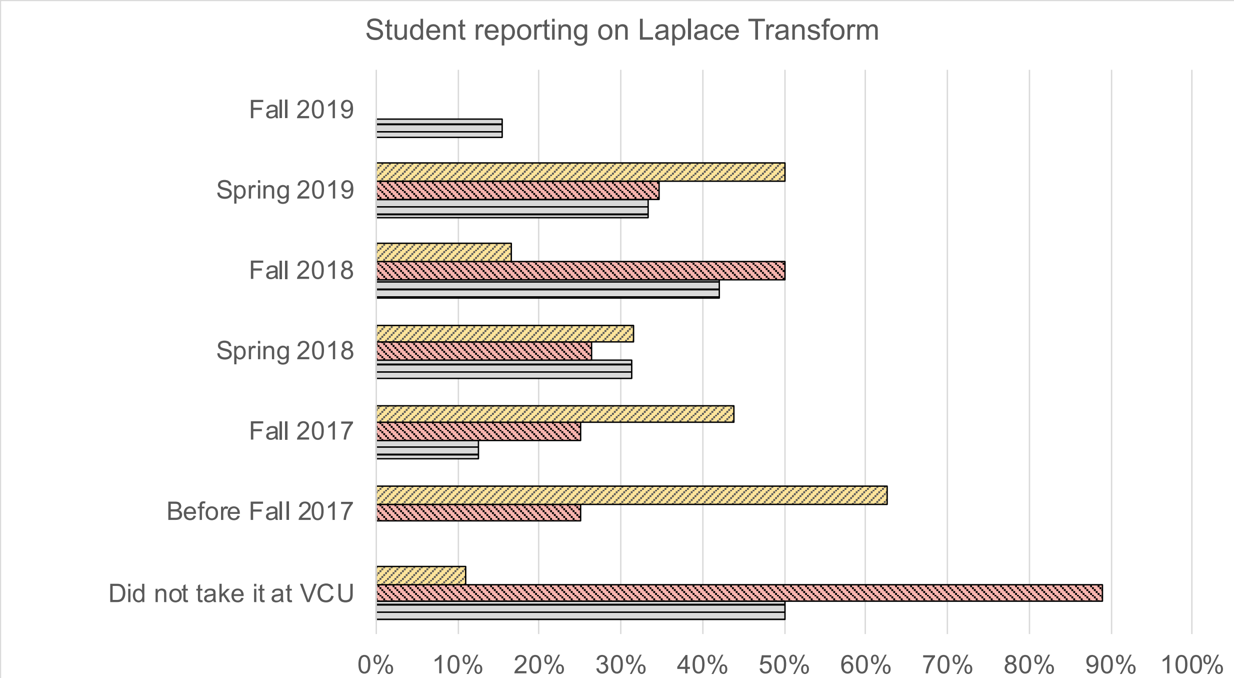
**Quantitative assessment:**

At this time, the assessment is being performed on all MATH 301 sections, with data being gathered as a baseline (from 2014 to 2017), midpoint (2017-2018) and full implementation (2019 and future). The data is being reviewed both from a year to year standpoint (are outcomes in engineering better post-2017 than they were pre-2017?) A table of engineering courses as well as their "distance" from MATH 301 has been developed and the data is being analyzed from a few perspectives:

1. Student outcomes in engineering vs outcomes in the math class.

2. Student outcomes in engineering vs the "distance" of the engineering class to the math class.

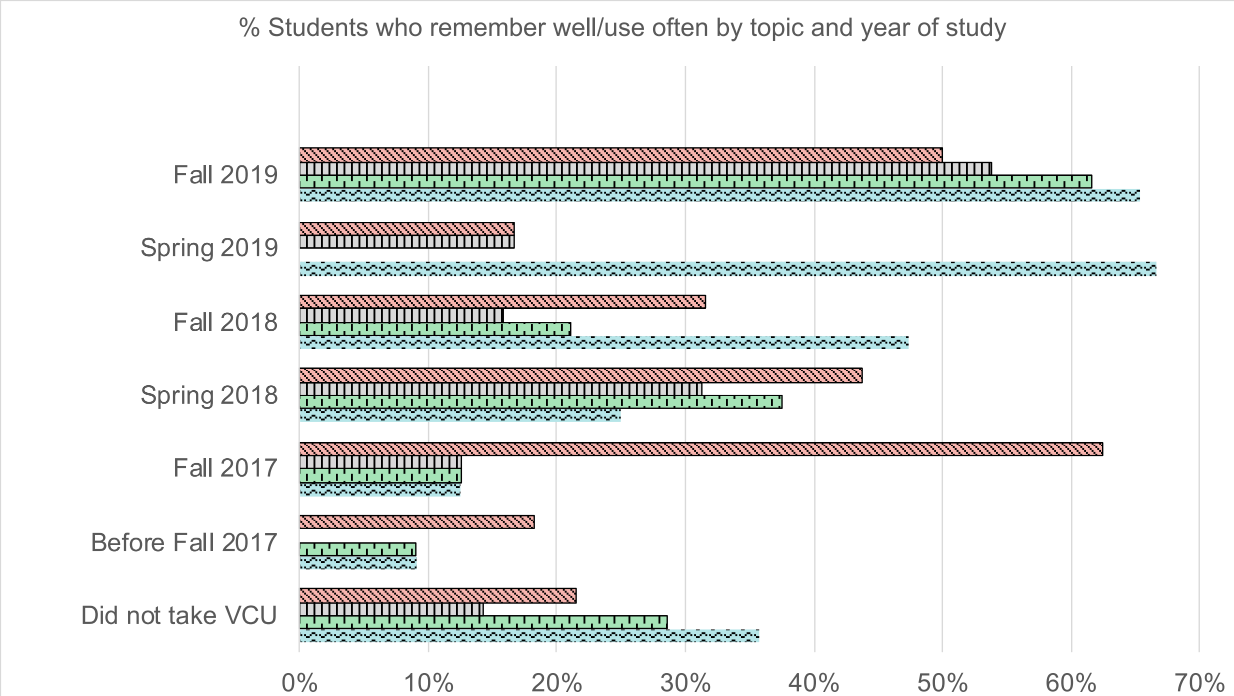
An example of how this data can be visualized is shown in Graph 7. This mosaic plot shows the dependence of the EGRB 215, "Computational Methods in Biomedical Engineering" grade to the MATH 301 grade. This course was chosen because it has Differential Equations as a pre-requisite and is placed, on the schedule, closest to the math course. This would allow us to capture at least some students from the newest cohort.




| | Did not take it at VCU | Before Fall 2017 | Fall 2017 | Spring 2018 | Fall 2018 | Spring 2019 | Fall 2019 |
|---|---|---|---|---|---|---|---|
| Remember well/use often | 11% | 63% | 44% | 32% | 17% | 50% | |
| Remember a little/use some | 89% | 25% | 25% | 26% | 50% | 35% | |
| Do not remember/do not use | 50% | 0% | 13% | 31% | 42% | 33% | 15% |

*Graph 5: Students self-reporting on their ability to recall and use Laplace Transform techniques and also the frequency with which they were asked to do so.*

Graph 7 shows aggregate results of the distribution of students between the years fall of 2015 – fall of 2019 who received a grade of A-F in the Differential Equations course (x-axis) against the corresponding % who received a grade of A-F in the subsequent course. So, for example, of all the students who received an A in Differential Equations, half received an A in Computational Methods (shown as red), a further 0.25 received a B (green), 17% received a C (blue) and the remainder received D and F. Correspondingly, for each grade on the x-axis, the width of the red square corresponds to the fraction of students who got a B in Differential Equations, while the height of the red square shows the corresponding fraction that got an A in Computational Methods. As expected, the number of students who get an A in Computational Methods decreases substantially as the math grade goes from A to F, although it doesn't go to zero. The data will be further analyzed on a per year basis, so see if these dependencies strengthen in the latter years. A significant amount of time before the project will be analyzed, to examine year-to-year variations.

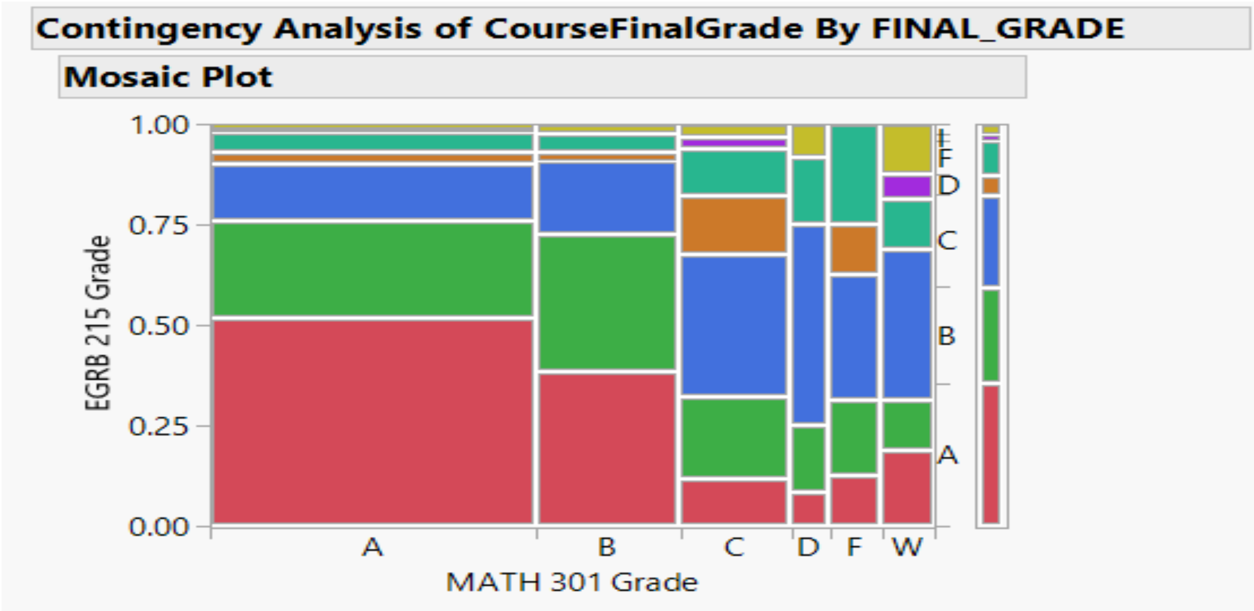



| | Did not take VCU | Before Fall 2017 | Fall 2017 | Spring 2018 | Fall 2018 | Spring 2019 | Fall 2019 | |
|---|---|---|---|---|---|---|---|---|
| Laplace transforms | 21% | 18% | 63% | 44% | 32% | 17% | 50% | |
| Variation of parameters | 14% | 0% | 13% | 31% | 16% | 17% | 54% | 0% |
| Undetermined coefficients | 29% | 9% | 13% | 38% | 21% | 0% | 62% | |
| Separation of variables | 36% | 9% | 13% | 25% | 47% | 67% | 65% | |

*Graph 6: Summary of all topics and all students arranged by year they took Differential Equations; % of students in each cohort who answered "Remember well/use often".*

In order for the plot in Graph 7 to be useful, we will need to track other factors that influence student performance and combine it to the assessment mechanisms described in the first paragraph of this section. In addition, a similar analysis will be performed on all classes with Integrated Equations as a prerequisite and when plotted against the student's delay between the math class and the corresponding engineering class, could provide insight on course sequencing and timing. A third analysis will be to map subsequent engineering classes to see if the students become more proficient in the math the more exposure they have to it in their engineering courses.

This analysis will have to be performed with the collaboration of the instructor of the course, as it must rely on targeted assessment exams that evaluate the student's specific skill set in the areas of interest.




*Graph 7: Mosaic plot comparing grades achieved in MATH 301 vs grades achieved in EGRB 215.*

**Lessons learned: Components of a systematic approach to collaborative, interdisciplinary teaching**

The VCU team (internal to one institution) as well as the larger team (comprised of 12 institutions) has compiled a list of attributes and habits that lead to strong inter-departmental collaborations in the area of teaching.

1. **Personal commitment**: team members are all internally motivated to make this collaboration work and are not anticipating specific personal benefit other than to see their students succeed and to "have fun". There is a sense of enjoyment of the activity, of anticipation of good outcomes for the student and the perception that this "needs to be done". This is not trivial, as this is the strongest component that the team has recognized keeps them going. The team is preparing to submit further proposals to support subsequent activities as well as further, more research-oriented activities. Absent funding, the impetus is there to continue activities in order to improve student outcomes and thus, student retention, and student and faculty satisfaction. There is support from the Deans' offices (math and engineering) for this initiative. One result is that the math department has agreed to teach separate sections of Calculus for engineering students in an attempt to facilitate implementation of this initiative in those classes as well.

2. **Intentionality**: the meetings, the reviews of work, the outreach to students and faculty are all necessary components to keep the collaboration relevant and active.

3. **A focus on action items**: previous attempts at collaboration were unsuccessful not because of lack of enthusiasm, but because of a lack of direction. The initial meeting between the math and engineering faculty leads and the determination of the action items

outlined in the beginning of this document were key to the successful collaboration between the principles.

4. **Inclusion**: actively try to grow the team within the institution and to include faculty in whatever way they can contribute to the project. This could include providing examples with solutions or merely copies of exams and homework. The meetings are crucial, as they not only allow the partner discipline to be specific about what they need from Math, but it also sensitizes the partner discipline to the challenges faced by faculty in the Math department.

5. **Accountability**: it is important to understand what each discipline is and is not responsible for. It is a definite outcome of this exercise that, while the Math department is enhancing their curriculum and streamlining it to the needs of the partner disciplines, the partner disciplines are also responsible for conducting targeted reviews of the specific material necessary for their courses. This project will allow us to open this dialog with the partner disciplines.

6. **Sociability**: by socializing engineering faculty with math faculty, we change the conversation from "us" and "them" to "we". This might be the most important outcome of this project.

**Conclusions:**

This paper described a systematic approach to developing inter- and cross-campus connections between faculty in math and partner disciplines to develop a scalable, transferable model for similar collaborations at other campuses. The University partners were purposefully diverse, of various sizes, geographic locations, diversity of offered disciplines and attitudes towards undergraduate education. They were similar in one aspect: they all had a math "champion" and a partner discipline "champion" who were recognized as being responsible for the success of the collaboration. A successful collaboration was judged through the ability of the team to:

1. Set up and maintain the collaboration at their own institutions: the VCU team maintained regular contact with each other and with the larger team (other math and engineering professors), participated in all the relevant conferences with the SUMMIT-P team and were able to report progress at all the project milestones.

2. Set up deliverables that were tailored to the needs of their own institution: through surveys, the fish bowl discussions, and further engaging engineering and math faculty in yearly meetings and reciprocal course visits, the team was able to develop substantive goals that were meaningful to both math and engineering. These methods can be modified and applied at any institution.

3. Maintain a high level of engagement: the VCU team has been continuously engaged in self-discovery, both within engineering and in math. Math has significantly modified their Integrated Equations course, not only to accommodate engineering but also to implement best practices: consistency between sections, common syllabus, active learning, and common exams. Engineering faculty have been incentivized to establish "Math bootcamps" (several classes in Electrical and Computer, Mechanical and Nuclear

and Chemical and Life Science Engineering) and are reviewing assessment to include more granular quantitative assessment of math concepts, and to include collateral concepts such as algebra.

4. Have a plan beyond the original grant: the VCU team has developed a determination to continue this project. The next deliverable is to finalize and optimize assessment of student outcomes and to develop a joint assessment mechanism between engineering and math. It is imperative, in engineering and in math, to improve retention and satisfaction, not just across the board, but specifically in student cohorts whose k-12 experience did not include advanced math classes and who will benefit the most from improved and targeted instruction and review mechanisms. In addition, engineering is planning on developing LONCAPA exercises to be used as "preview" practice problems in math and "review" problems in engineering.

The partnership has been established at VCU between math and engineering. Initial assessment of the to-date activities indicates that the amendments to math and to engineering are leading to positive results. Now that the partnership has been established, it is imperative to continue and expand on the original work to include robust assessment of student outcomes with direct links to math and engineering activities that are developed to improve student outcomes. The assessment is key to determining which activities are the most impactful and lead to the best outcomes.

**List of Citations:**

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